Re: Largest Set in ZFC?

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- *From*: "Peter Webb" <webbfamily@xxxxxxxxxxxxxxxxxxxxxxxxxx>

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<SNIP>

> but it seems there is a
> more fundamental reason for the vagueness of the definition of X. If anybody
> could produce a specific such X, then they would have a model of ZF (a
> collection of sets that satisfy ZF). This would prove the consistency of ZF.
> AFAIK, there is no theoretical reason why such a set X could not be
> described, but none ever has, and its seems pretty obvious that none ever
> will. Hence the requirement to vague it up a bit.

We can derive omega from any set that satisfies AoI.
AoI says a set exists. It doesn't say more than one
such set exists. Since omega is defined as the "smallest"
set satisfying AoI, I see no reason to assume any
other set satisfies AoI.

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Well, how about {w U {w}}, aka S(w)? Or are you arguing that it is impossible to prove w <> S(w) ?

Your argument suggests omega is a model of ZF.
Can't we prove omega exists in ZF?

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Yes but No. Omega is not X, because it doesn't include w+1, and hence doesn't satify the requirement that "and whenever y is in X, so is S(y)". As I indicated, nobody has ever come up with an actual set X that meets the requirements of the definition. If they did, they would prove the consistency of ZF by providing a "model" of ZF(C), viz the set X itself.

I don't see how ZF can have a set with greater
cardinality than omega (without using Powerset).

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Nor can I, and nor apparently can anybody else in this thread, although we all seem to agree its probably true.
No PA, no c.

You might find this interesting − its only tangentially related, but touches on a possible model for X (X=V=L).

http://en.wikipedia.org/wiki/Axiom_of_constructibility

Unfortunately, there is still no actual construction of X that we can perform without using transfinite recursion, and my PC doesn't do that.

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