

Re: Largest Set in ZFC?

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- *From:* MoeBlee <jazzmobe@xxxxxxxxxxxx>
 - *Date:* Tue, 11 Mar 2008 17:33:24 -0700 (PDT)
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On Mar 11, 4:10 pm, "Peter Webb" <webbfam...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

First, your quoting format is all messed up. Quotes of mine are appearing as if quotes of yours. And your line of asterisks approach to separating falls apart when I'm REPLYING. Can you do anything about that in future posts please?.

Second, please read my post to the end before responding paragraph by paragraph, as you will see that the latter part of my post explains further about the early part. By the end of my post you should see the mistake(s) you're making.

Most basically, a model of ZF is not just a set that "satisfies the axioms" but rather a set (the universe of the model) AND a RELATION on that set such that the set AND the RELATION satisfies the axioms.. And for some models of ZF, that relation might NOT be the membership relation on the set.

"MoeBlee" <jazzm...@xxxxxxxxxxxx> wrote in message

news:874b4355-ca2f-4e3b-acfc-e4ceac9397f1@xx

On Mar 9, 1:55 am, "Peter Webb" <webbfam...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:

Your [reaste's] argument suggests omega is a model of ZF.

No, that's wrong. In ZF we prove that there exists a unique least inductive set, which we name 'w'. That doesn't entail that in ZF we prove w is the universe of a model of ZF. Rather, in ZF, we prove that IF ZF has a model then w is the universe of a model of ZF.

That's my quote above.

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Well, w is not a model of ZF. There are sets constructible from w which are not within w . Like $w+1$, for example. w cannot be a model of ZF because you can construct sets in ZF that are not a subset of w .

That's your quote above. .

Anyway, no, w IS a universe of a model of ZF. That follows from the strong form of the Skolem theorem. You're confused about what is required for a set to be a universe of a model. Yes, in ZF we prove that there are sets that are not members of w , but that does NOT preclude that w is a universe of a model. And the reason is that, for example, the term ' $w+1$ ' does not have to map to $w+1$ per the model.

w is a model of ZF–AoI, excepting for the slight technical problem you can't prove the existence of w without AoI.

Your quote above.

It is correct that ZF–I is undecided as to the existence of an infinite set. And it is correct that w is a universe of a model of ZF–I. But w is ALSO a universe of a model of ZF.

From our exchange I can see that you have a misconception about what a model is. Your misconception reveals itself in your notion that that every set that ZF proves to exist must be a member of the universe of any model of ZF.

Also, please look up the versions of the Skolem theorem that includes this:

If a first order theory has a model with an infinite universe, then that theory has a model with w as its universe. From that you see that if ZF has a model then ZF has a model in which the universe is w .

In a similar but not identical way, the set of all ordinals would be a model of ZF, but unfortunately we know this doesn't exist.

You can have a "proper class" of all ordinals, but unfortunately the definition calls for a "set" X (and quite reasonably; classes are not even defined in ZF).

Your quote above.

That in ZF there is not a set that has all ordinals as members does not detract from the fact that w is the universe of a model of ZF.

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Yes but No. Omega is not X, because it doesn't include $w+1$, and hence doesn't satisfy the requirement that "and whenever y is in X, so is $S(y)$ ".

You're TERRIBLY confused about this. To be successor inductive it is not required that the set have w as a member so it is not required that the set have $w+$ as a member. What is required is that IF w is a member then $w+$ is a member; but it is not required that w is a member.

My quote above.

Unless we have a terminology problem, what you are saying is wrong.

Your quote above.

No, what I said is perfectly correct.

I said that there is no known set of sets which which meets the definition of X.

Your quote above.

No, that's not what you said. What you JUST said is something quite different. And as to what you JUST said, yes, I believe you are correct that there is not a set that has as members every successor-inductive .

If by w and omega we are referring to the smallest infinite ordinal, then clearly w cannot be X, because w doesn't include $w \cup \{w\}$ which is one of the explicit requirements of X and indeed of the AoI.

Your quote above.

No, you terribly misunderstand the axiom of infinity. The 'X' we've been talking about is a successor inductive set; it's NOT, nor is it supposed to be, a SET Of successor inductive sets. Somehow you "jumped up" a level. You need to get back down to the axiom which asserts the existence of a successor inductive set and that has nothing to do with any set OF successor inductive sets (indeed, as we agree, there is no set that has as every successor inductive set as a member).

And so the axiom of infinity does NOT require that a set have the successor of w as a member.

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The axiom of infinity simply asserts that there exists a set that has 0 as a member and has the successor of any member as a member. So if w itself is not a member then the successor of w doesn't have to be a member. It's only if w is a member that the successor of w has to be a member. And the LEAST successor inductive set, which IS w , is one that indeed does NOT have w as a member. There are OTHER successor inductive sets (but they're not the least successor inductive set) that have w as a member and so have the successor of w as a member, but, again, none of those is the LEAST successor inductive set.

Can you name a single set X that meets the requirements of the definition of X ? I can't, and AFAIK nobody has any idea of such a set.

Your quote above.

If by ' X ' you mean a SET of successor inductive sets, then of course, there seems not to be one (I bet there's a simple proof that there is not one, probably an argument that if there were one then there'd be a set of all ordinals). But ' X ' originally in this discussion was not for a SET of successor inductive sets, but rather for a successor inductive set.

That is why the definition of X is left vague – If we could produce an actual set X which satisfied the definition of X and all the other axioms of $ZF(C)$, we would have a model of $ZF(C)$, which by Godel's completeness theorem would prove the consistency of $ZF(C)$ from within $ZF(C)$. This is not impossible, but after a century of looking, nobody has found one, and it looks pretty unlikely such a set X will ever be found.

Your quote above.

But in this discussion earlier the question was not leaving ' X ' vague in the sense of X as a SET of all successor inductive sets, but rather of X as a successor inductive set. X is a successor inductive set. Get rid of this notion that ANYONE purports that X is a SET of all successor inductive sets or that X is a set of even SOME successor inductive sets (though in the case of some, we can go on to prove that). So one reason we may say the axiom does not further specify as to X is that we don't NEED to further specify, since the specification we do desire comes from separation and extensionality.

As

I indicated, nobody has ever come up with an actual set X that meets the requirements of the definition.

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What are you talking about?! We DO prove the existence of a unique defined set that is successor inductive.

My quote above.

Show me a proof that there is a set X with the properties described in the OP's textbook.

Your quote above.

What textbook is the OP's textbook?

And again, of course I'm not going to try to prove there is a SET of all inductive sets. That shouldn't even have been dragged into the conversation.

What exists is a successor inductive set. That comes from the axiom of infinity, since that IS what the axiom of infinity asserts. Then there is a unique LEAST successor inductive set, which comes from separation and extensionality.

Even better, as the OP was complaining that the definition of X is vague, how about producing a single example of such a set? w doesn't work, because it doesn't include $S(w)$, which is an explicit part of the definition of X.

Your quote above.

NO! Again, X is not a SET of successor inductive sets. X is just a successor inductive set. But it's bound by an existential quantifier, so we don't know yet that there is a unique LEAST successor inductive set. But we get that right away anyway from separation and extensionality, then name the set 'w'.

And by the Skolem theorem we also get that w is the universe of a model of ZF. That seems like it provokes a plain contradiction ONLY if you have a misconception as to what a model is.

If they did, they would prove the consistency of ZF by providing a "model" of ZF(C), viz the set X itself.

No, that is wrong, as I explained earlier in this post.

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My quote above.

I can't see this. Any set X constructible within ZF that satisfied all of the axioms of ZF (including AoI) would be a model of ZF. Agreed?

Your quote above.

And no, you're leaving out A KEY INGREDIENT.

A model of ZF is not just a set X but a set X and a relation R on X such that $\langle X, R \rangle$ satisfies the axioms.

And ZF does not prove that there exists such an $\langle X, R \rangle$.

A model of ZF
would prove the consistency of ZF (by Godel's completeness theorem). Agreed?

Your quote above.

Of course.

The consistency of ZF has never been proved using ZF alone. Agreed?

Your quote above.

Of course.

Therefore there is no known set X . Agreed?

Your quote above.

There is no $\langle X, R \rangle$ that ZF proves to satisfy the axioms of ZF.

If you think you have a set X , and can prove it meets all the axioms of ZF,
please post it.

Your quote above.

No, a set alone doesn't meet the axioms, but rather a set and a

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RELATION on that set.

And what you're overlooking is that R does not have to be the membership relation on the set. A model may map the 'e' symbol of the language of set theory to a relation OTHER than the membership relation on the universe of the model. And that has ramifications down the line so that, e.g., 'w+1' might not map to $w+1$ because all those defined terms such as 'w+1' got defined in terms of 'e' while 'e' itself might not map to membership itself.

If you cannot name even a single such set X (as I suspect) the reasons that the definition of X is pretty vague should be obvious – nobody can describe even a single, concrete set X which meets the criteria of ZF.

Your quote above.

No, as you've should see now as I've explained in tedium..

MoeBlee