Re: Godel's comments about the "true reason" for incompleteness

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- *From*: LauLuna <laureanoluna@xxxxxxxx>
- *Date*: Fri, 14 Mar 2008 01:06:24 −0700 (PDT)

On Mar 13, 2:43 am, Newberry <newberr...@xxxxxxxxx> wrote:

> On Mar 12, 1:37 pm, LauLuna <laureanol...@xxxxxxxx> wrote:
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>> On Mar 12, 4:39 am, Newberry <newberr...@xxxxxxxxx> wrote:
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>>> On Mar 11, 4:30 pm, LauLuna <laureanol...@xxxxxxxx> wrote:
>>>
>>>> On Mar 11, 6:47 pm, djr...@xxxxxxxxxx wrote:
>>>>
>>>>> "The true source of the incompleteness attaching to all formal systems of mathematics, is to be found−−−as will be shown in Part II of this essay−−−in the fact that the formation of ever higher types can be continued into the transfinite (c.f., D. Hilbert, 'Über das Unendliche', Math. Ann. 95, p. 184), whereas in every formal system at most denumerably many

types occur. It can be shown, that is, that the undecidable propositions here presented become always become decidable by the adjunction of suitable higher types. A similar result also holds for the axiom system of set theory."

This comment by Godel has me confused, first of all by what he means by "true source". Isn't his proof and later refinements/ generalisations of it a "true source" for incompleteness? Also, I was under the impression that the whole point of Godel's theorem is that any kind of proof procedure or list of proof procedures that you can *even indicate* will not be able to decide all mathematical propositions. It sounds as if he is saying "ah, we can just continue adjoining higher types in such and such a manner, and eventually arbitrary statements become decidable (i.e. for any statement, it eventually becomes decidable)". I thought the whole point of Godel was that even if you spent a billion years outlining precisely a method of coming up with formal systems, there would still be propositions that could not be resolved by any of those formal systems.

> > This is an interesting topic. Among other reasons because Gödel never
> > gave a proof, for all I know, although he mentioned the point later
> > again.
>
> > Gödel stated that higher types with the corresponding comprehension
> > axioms render the undecidable sentences decidable (though, of course,
> > there are also undecidable sentences in the resulting systems).
>
> > I wonder how exactly this is related to the following:
>
> > 1. Only an enumerable number of sets of naturals are definable in a
> > formal system whereas P(N) is not enumerable.
> > 2. Gödel stated elsewhere that the ultimate cause of arithmetic
> > incompleteness is the fact that arithmetic truth is not arithmetically
> > definable (Tarski 1933); it's natural to presume he had in mind some
> > relationship between the two phenomena.
>
> The reason for incompleteness is that bivalence is inadequate.− Hide quoted text −
>
> − Show quoted text −

How so?

For the same reason "this sentence is not tue" has neither of the two

values. Furthermore the so called "vacuously true sentences" are
neither true nor false. Goedel's formula is vacuously true

~(Ex)(Ey)(Pxy & Qy) (1)

Let m be the Goedel number of (1), and Qy is satsfied only by y = m.
If

~(Ex)Pxm

then (1) is vacuously true. If vacuously true is actually not true
(and not false either) then (1) does not have a truth value. This of
course is not the case in classical logic. A non−bivalent logic is
required, and that is basically what Goedel's theorem tells us.− Hide quoted text −

− Show quoted text −

Under the conditions you propose, your second sentence implies (1).
But this does not mean that any of them is vacuously true.

Gödel's sentence G is, standardly interpreted, a sentence about
naturals. How could it lack any truth value? Either there is a natural
satisfying the predicate involved or there is not such.

And what about other undecidable sentences, as those expressing
consistency?

Regards
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