Re: Set theory and identity theory

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On Fri, 14 Mar 2008 07:13:25 −0700 (PDT), MoeBlee <jazzmobe@xxxxxxxxxxx>
wrote:

> > I don't think so. Note that here "x = y" just "means" that Az(zex <−> zey).
>
> Sure. But since 'e' is the only primitive, all other properties are "generated" by it.

Yes, of course. In our theory. (But that does not take over to our model. There sets, i.e. the objects in our model, might have additional properties, say, colors.)

> So, I was thinking (not in a rigorous, but rather in heuristic or speculative way)

Of course. Same with me (here).

> that perhaps the fact that the sole primitive 'e' now "controls every other property down the line" provides us a with a kind of "end run" around the problem that we can't state the identity of indiscernibles in first order (not even in the meta−theory), so that that would lead to some way to show that our axioms of set theory are only satisfied by '=' getting mapped to the identity relation on the universe.

Maybe my grasp of the English language isn't that good... :−/

> But perhaps not.

Didn't you like my considerations?

.... consider a universe (for our model) which contains "decorated" sets (or "colored" sets if you like. Just assume that we have objects which not only contains elements, but in addition have a color). Then we might have two _different_ decorated sets a, b which just have the same elements, but different colors. In this case we would have

Az(zea <−> zeb).

And hence

a = b

would be satisfied, though a and b would NOT be identical (in the usual sense of the word).

> So what puzzles me now is when someone says something like "consider a model of ZF", how do I know whether the person intends that '=' is treated as from identity theory and with the ordinary fixed semantics so that the model must map '=' to the identity relation on the universe or whether the person is taking '=' as defined, thus without the fixed semantics, so that '=' might not map to the identity relation on the universe?

Right! Hence I actually prefer to develop ZFC in the framework of /FOPL with identity/. (And you will see that MOST authors also prefer that approach).

> This ambiguity is real since it's usually the case that we talk about set theory without being so specific about where our '=' came from, whether from identity theory and its fixed semantics or from definition from the sole primitive 'e'.

Not in my case. I actually always (unconsciously) assume that we are working with ZFC in the framework of /FOPL with identity/. Actually the cases (textbooks) where this is not the case are rather rare, I'd guess.

IMHO (even not taking into account the considerations from above) there's a REASON why considering "identity" a logical primitive (in FOPL with identity).

Of course, in a system of 2OL we actually might _define_ identity (for objects/things):

a = b :<−> AF(F(a) <−> F(b))

(And of course if we define "=" in set theory, we TRY something analogous; but −as you know− set theory is a first−order theory, and hence not powerful enough to characterize identity, etc. −>Skolem.)

F.

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