Re: Set theory and identity theory

Source: http://sci.tech−archive.net/Archive/sci.logic/2008−03/msg00936.html

- *From*: MoeBlee <jazzmobe@xxxxxxxxxxx>
- *Date*: Fri, 14 Mar 2008 08:08:23 −0700 (PDT)

On Mar 14, 7:39 am, G. Frege <nomail@invalid> wrote:

> On Fri, 14 Mar 2008 07:13:25 −0700 (PDT), MoeBlee <jazzm...@xxxxxxxxxxx>
> wrote:
>
> > > > I don't think so. Note that here "x = y" just "means" that
> > > > Az(zex <−>
> > > > zey).
> >
> > > Sure. But since 'e' is the only primitive, all other properties are
> > > "generated" by it.
>
> Yes, of course. In our theory. (But that does not take over to our
> model. There sets, i.e. the objects in our model, might have additional
> properties, say, colors.)

But if our model theory is itself given within a meta−theory that is a
set theory (which I take it to be), then ain't no properties 'ceptin
those made up from 'e'.

> Didn't you like my considerations?

Yes, and they are along the lines I've pretty much always understood.
I just want to be more clear that my reasons aren't enough to overide
the considerations you mentioned.

> ... consider a universe (for our model) which contains "decorated"
> sets (or "colored" sets if you like. Just assume that we have objects
> which not only contains elements, but in addition have a color). Then we
> might have two _different_ decorated sets a, b which just have the same

elements, but different colors. In this case we would have

Az(zea <−> zeb).

And hence

a = b

would be satisfied, though a and b would NOT be identical (in the usual sense of the word).

Yes, I do understand your reasoning. Except, as I mentioned, if the the model theory itself is in set theory, then epsilon is the only primitive property even at the meta−level.

So what puzzles me now is when someone says something like "consider a model of ZF", how do I know whether the person intends that '=' is treated as from identity theory and with the ordinary fixed semantics so that the model must map '=' to the identity relation on the universe or whether the person is taking '=' as defined, thus without the fixed semantics, so that '=' might not map to the identity relation on the universe?

Right! Hence I actually prefer to develop ZFC in the framework of /FOPL with identity/. (And you will see that MOST authors also prefer that approach).

It seems most authors do (but it's not entirely clear in many cases), but some authors specifcally do not. So I'm wondering whether the authors that do not expect that it is implicit that they too are using a fixed semantics for '=' even though their '=' doesn't come from identity theory?

This ambiguity is real since it's usually the case that we talk about set theory without being so specific about where our '=' came from, whether from identity theory and its fixed semantics or from definition from the sole primitive 'e'.

Not in my case. I actually always (unconsciously) assume that we are working with ZFC in the framework of /FOPL with identity/. Actually the cases (textbooks) where this is not the case are rather rare, I'd guess.

They're not usual, but not so very rare I don't think. Anyway, my point is that we may need to check our tacit assumptions in this regard, especially since it is not very common for a set theory author to explictly mention that the semantics for '=' is the fixed one.

> IMHO (even not taking into account the considerations from above) there's a REASON why considering "identity" a logical primitive (in FOPL with identity).

I agree that there is a good reason for taking set theory as an extension of identity theory. But still it's a matter of preference.

> (And of course if we define "=" in set theory, we TRY something analogous; but −as you know− set theory is a first−order theory, and hence not powerful enough to characterize identity, etc. −>Skolem.)

Do you know where I can get a FORMAL working out of the result about not characterizing identity? I don't dispute the result; I do understand why the identity of indiscernibles is not GENERALLY expressible for first order; but I wonder whether the consideration of a finite number of primitives provides some kind of special case.

MoeBlee

.