

Re: Gödel's comments about the "true reason" for incompleteness

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On Mar 14, 1:06 pm, LauLuna <laureanol...@xxxxxxxxx> wrote:

[...]

Gödel's sentence G is, standardly interpreted, a sentence about naturals. How could it lack any truth value? Either there is a natural satisfying the predicate involved or there is not such.

And what about other undecidable sentences, as those expressing consistency?

There are no such sentences, at least not formally expressible ones. No sentence about natural numbers can ever tell you something about "all" sentences of Peano Arithmetic or of whichever theory whose consistency you are considering. The coding employed by Gödel to make this interpretation is not justifiable at first order. Here is the simple argument. Consider a proposition like

$P \& \sim P \rightarrow Q$

This actually becomes a sentence of, say, PA, only after we replace the sentential variables P and Q with specific propositions about natural numbers. Therefore there cannot be a single proof of $P \& \sim P \rightarrow Q$ within PA, even though it is considered a basic logical tautology. Without any coding, there are actually infinitely many proofs in PA of infinitely many propositions of the form $P \& \sim P \rightarrow Q$.

But with Gödel's coding, a notion like "From a contradiction $P \& \sim P$, any sentence Q (in the language of PA) follows" becomes expressible, via coding, as sentence S of PA. Now by what I said in the previous paragraph, S , in so much as it encodes $P \& \sim P \rightarrow Q$ for an arbitrary P and an arbitrary Q , cannot have a single proof in PA. But S cannot have a refutation in PA either, for such a refutation would imply there there

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exists a contradiction of the form $P \& \sim P$ from which some specific sentence Q is not deducible, which contradicts *ex falso quodlibet* and is not poaaible. Therefore we conclude that S has to be undecidable in PA . But that would imply, by the completeness theorem, that there would have to be a model of PA in which S is false and such a model cannot exist.

The problem here is in the coding that allows us to express $P \& \sim P \rightarrow Q$ as a single sentence in the language of PA . The only way to maintain consistency is to deny the existence of any such coding, in order to avoid the objection raised above. Such a coding reaches out beyond what is acceptable at first order with fatal consequences. $P \& \sim P \rightarrow Q$ should actually express a metamathematical truth about PA and can never be formally admitted as a sentence in the language of PA via coding (e.g. of the kind employed by Godel).

I raised this objection to Godel's theorems in the following thread in sci.logic and did not receive any reply from the Godelians:

<http://groups.google.la/group/sci.logic/msg/cc1af3d1f93ca3f1>

To not answer this and the whole gamut of objections to Godel, Cantor, Einstein et al. that is inherent in the logic $NAFL$ is, of course the prerogative of Godelians, Cantorians, Einsteinians, etc. After all, there is no way for a powerless individual like me to change human nature. But sooner or later, the farcical nature of this state of affairs will become clear to everybody and hopefully some one of a suitable stature and integrity will break away from the status quo and seriously consider my contribution.

Regards, RS

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