Re: Godel's comments about the "true reason" for incompleteness

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- *From*: "R. Srinivasan" <sradhakr@xxxxxxxxxx>
- *Date*: Thu, 20 Mar 2008 10:55:13 −0700 (PDT)

On Mar 19, 10:16 pm, MoeBlee <jazzm...@xxxxxxxxxxx> wrote:

> On Mar 19, 12:09 am, "R. Srinivasan" <sradh...@xxxxxxxxxx> wrote:

[...]

>> Godel translated a proposition of the form P&~P−>Q into a proposition
>> in the *object* language of arithmetic, say, PA.

> What specific formula of the meta−language and its specific
> translation into the language of PA are your referring to?

Why don't you ask this question to the Godelians? All I am interested
in at this point is that there is such a formula S in the language of
PA.

>> Call this proposition
>> S. Note that S is a specific propositon involving only numbers.

> What does "involving only numbers" mean?

Means it is a formula in the language of PA.

>> By your own assertion above you do not admit P&~P−>Q in the object
>> language of arithmetic. So if you scan all proofs of PA, you will not
>> find a proof of P&~P−−>Q (a proof must end with the propositon
>> proven). You will not find a refutation of P&~P −−> Q either in the

list of PA−proofs. For if you did, then P&~P&~Q must hold in every model of PA, for some specific (number−theoretic) propostions P and Q and we know that is not possible.

No, the reason we don't have a refutation in PA is that '(P&~P) −> Q' is not in the language of PA. It's in the meta−language.

Wrong. For PA to refute (P&~P)−>Q, PA needs to prove (P&~P&~Q) for *one* specific P and *one* specific Q, or, what amounts to the same thing, PA needs to prove a contradiction. E.g. if PA proves

GC & ~GC & ~TPC

then PA has refuted (P&~P)−−>Q. Here I am taking 'P' as GC (Goldbach's conjecture) and 'Q' as TPC (twin prime conjecture). But P and Q could be something very simple, say, 2+2=4 and 2+2=5.

You can immediately see that PA would refute P&~P−>Q if PA proves a contradiction. But PA does not have the expressive power to prove P&~P−

Q. Of course, I can now sense that you are going to jump up and down

insisting that PA has not refuted P&~P−>Q even if PA is inconsistent. Sure, if P&~P−>Q is not in the language of PA, its formal negation is also not in the language of PA. But all I am saying here is that PA can prove itself inconsistent by proving a contradiction. You don't have to encode the notion of consistency into PA in order for PA to prove itself inconsistent. In which case PA would have refuted P&~P−>Q in substance.

To see an analogy, it could be that FLT is not expressible in the language of some theory T of arithmetic if you don't permit exponentiation. But T could still prove ~FLT, by construction, if there exist specific positive integers x, y, z and n>2 such that....... Do you get it?

You can also see that P&~P−>Q *is* actually a consistency statement for PA. Assuming you are a staunch and diehard Godelian, you should now be able to get an intuitive feel for why if 'P&~P−>Q' is translated into a proposition S in the language of PA, then S has to be undecidable in PA. For that matter even ~(P&~P) or Pv~P, etc are consistency statements for PA which are not expressible in the language of PA and which cannot be proven in any first−order theory.

Now Godel translated the meta−theoretical propostion P&~P−>Q into a specific number−theoretic propostion S in the language of PA. Note

> that S is not a variable; it is a constant and we have a construction
> for S.

So I take it 'S' is a defined constant in the meta−language.

> Since we just now argued that P&~P−−>Q is not either provable
> or refutable in PA,

Because it's not even in the language of PA.

> it follows that S must also be undecidable in PA.

No, because S is in the language of PA. 'S' is a defined constant of the meta−language. But S is a formula of the object language PA.

> But this means that there must exist a model of PA in which S is
> false, and such a model cannot exist, for the same reason argued
> earlier. This is a contradiction.

Wow! What a completely amateurish confusion you have! You're competely mixed up in use/mention and meta−language/object language. If a formula is not in the language of theory, then it's not a theorem of the theory, but that doesn't mean there are models for the language in which the formula is false. Only formulas of the LANGUAGE have values of 'true' or 'false' in models for the language. You're blatantly conflated S with '(P&~P) −> Q'. They're in different languages. That '(P&~P) −> Q' is not a theorem of PA doesn't entail that S is not a theorem of PA, even though S is a "translation of '(P&~P) −> Q' .

Of course I know what you have said above, and I have explicitly stated it in a subsequent post. I have already said that it is a *requirement* of first−order logic that a sentence like P&~P−>Q should not be expressible in any first−order language, which will not allow quantification over formulas.*That* is why we do not require that a theory like PA should have a model in which P&~P−>Q is false (even though it is not provable in PA) because such a formula is not admitted in the language of PA.

Now think of the following situation. Let us add P&~P−−>Q (and only this single sentence, where P and Q are arbitrary) into the language of PA, but let us not change the principles of proof or expand the logic in any other way. Then we would have to accept that P&~P−>Q is undecidable in PA and there must exist a model of PA in which it is false, which is not possible. That is why a sentence like P&~P−>Q *should* never be expressible in any first−order language; if it were, inconsistency will follow.

> > In fact my understanding is that PA proves S. If the meta−theoretical sentence P&~P−>Q does translate to S and if this coding is accepted as legitimate, this is tantamount to the claim that PA proves P&~P−−>Q,
>
> NO! And that's the exact contrapositive of what I just mentioned. PA does not prove '(P&~P) −> Q' because '(P&~P) −> Q' is NOT IN THE LANGUAGE of PA, but S IS in the langauge of PA, so PA may very well prove S while OF COURSE it does not prove '(P&~P) −> Q' .

Sure, sure. I now see that you are the mother of all formalists. If P&~P−>Q has been translated to a formula S in the first−order language of PA, and if PA proves S, are you denying that PA has *effectively* proven itself consistent? Even by your GODel, PA is inconsistent. Now I see that I have waved the red flag and you are going to charge with further futile formlastic arguments for why that is not the case.

But I don't need Godel to make that argument. It is a requirement of first−order logic that such a sentence cannot be translated into a first−order language and we have started out by formulating the logic that way. We have already fixed that while every instance of P&~P−>Q is provable in first−order theories, P&~P−>Q itself is not provable for arbitrary P and arbitrary Q.; Admitting such a sentence into a frist−order language, whether directly or by translation, will lead to inconsistency because the logic would have been expanded beyond the rules we ourselves have laid down. *If* such a sentence S (to which P&~P−>Q has been translated) exists in the language of PA, then S *has* to be undecidable in PA in order to remain consistent with the way we have formulated first−order logic. But such undecidability is not possible.

And just out of curiosity, why do even Godelians say things like "PA can prove itself inconsistent", etc., After all, by what you have said above, "PA is consistent" is not formally a sentence in its language. Are these people talking rubbish?

Regards, RS

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