Re: Godel's comments about the "true reason" for incompleteness

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- *From*: "R. Srinivasan" <sradhakr@xxxxxxxxxx>
- *Date*: Fri, 21 Mar 2008 02:53:58 −0700 (PDT)

On Mar 21, 3:45 am, MoeBlee <jazzm...@xxxxxxxxxxx> wrote:

> Re: Mar 20, 10:55 am, "R. Srinivasan" <sradh...@xxxxxxxxxx>:
>
> After having responded to that post, I think it would help to focus on just, say, two of the most obviously ridiculous things in it:
>
>> Now think of the following situation. Let us add P&~P−−>Q (and only this single sentence, where P and Q are arbitrary) into the language of PA,
>
> Please get back after you've consulted a logic book to see that it makes no sense to say that we add just a single sentence to a language.

Of course I said "inconsistency will follow". Why do you make such a big deal of this? I tried to make the point that what Godel did amounts to precisely what you are ridiculing. He brought in a sentence like P&~P−>Q (note: I am talking about the sentence with P and Q as variables) into a first−order language via translation, call this sentence S say, in the language of PA. Note again: Godel translated the *schema* P&~P−>Q into a single sentence in the language of PA. But the logic is still the same, the proof principles are still the same. So such a sentence should be neither provable nor refutable in any consistent first−order theory (using only finitary principles), including PA. *I* am asserting that as self−evident. No first order theory that uses only finitary principles can ever establish the truth of a schema via a single proof, either directly or indirectly (e.g via Godel's translation procedure).

If you want to claim that PA proves S and S is the translation of the *schema* P&~P−>Q, then you have *obviously* stepped beyond the boundaries of first−order logic at the translation stage. I am rejecting the translation procedure employed by Godel as infinitary (there are other reasons for this as well).

By the way, what precisely is the metatheory in which this translation is carried out? My understanding is that Godelians claim that this metatheory also uses nothing stronger than finitary reasoning which can be carried out in a weak theory of arithmetic. That is as good as claiming that P&~P−>Q is a legitimate sentence in the language of such a theory (say, PA), as *proven* by Godel's translation procedure carried out using nothing stronger than PA. I do not accept this because in my view such a translation must use infinitary priciples that *obviously* step beyond the boundaries of first−order logic, since such ;principles have managed to legitimize a sentence that is out of bounds for a first−order language.

> If
> P&~P−>Q has been translated to a formula S in the first−order language of PA, and if PA proves S, are you denying that PA has *effectively* proven itself consistent?

> Please get back after you've consulted a book on logic to see that a theory proving a tautology (or even just a contingency) does not entail that the theory is consistent, and not anywhere in the imagination of anyone but the most bitterly confused does it entail that the theory has ""effectively" proven ITSELF consistent".

Again: No first−order theory ever proves a tautlogoy. That is your misunderstanding. There is no such thing as *a* tautology as far as first−order languages are concerned. First order languages can only recognize instances of a schema representing a tautology. The tautology itself, say, P&~P−>Q where P and Q are variables, is NOT provable in any first order theory, at least not until Godel came along and managed to translate a *schema* P&~P−>Q into a single sentence S in the frst−order language of arithmetic (say, PA).

I am asserting that the sentence P&~P−>Q (note that this is not a first−order sentence, it has variables in it) or say, ~(P&~P), etc., which are tautologies, are actually metamathematical truths with respect to first order theoreis (say, PA) that prove every instance of the corresponding schemas, *assuming* that such theories are consistent. The only way for a first−order theory (say, PA) to refute a tautology (note: the sentence with variables in it) is by proving a contradiction, and that is possible only if such a theory is inconsistent. That is why I assert that a sentence like P&~P−>Q, with variables in it, is a consistency sentence and if Godel managed to translate this sentence into a single sentence S in the language of a first order theory (say, PA) then S is a consistency sentence for PA as far as I am concerned.

You don't even begin to understand what I am talking about here. Try again.

Regards, RS
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