Re: Godel's comments about the "true reason" for incompleteness

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- *From*: "R. Srinivasan" <sradhakr@xxxxxxxxxx>
- *Date*: Fri, 21 Mar 2008 15:24:08 −0700 (PDT)

On Mar 21, 11:20 pm, MoeBlee <jazzm...@xxxxxxxxxxx> wrote:

> On Mar 21, 1:45 am, "R. Srinivasan" <sradh...@xxxxxxxxxx> wrote:
>
> > > > Now think of the following situation. Let us
> > > > add P&~P−−>Q (and only
> > > > this single sentence, where P and Q are
> > > > arbitrary) into the language
> > > > of PA,
> >
> > > That makes NO SENSE! You don't add just a SENTENCE to
> > > a language.
> >
> > Of course you can.
>
> Not for ordinary first order languages such as we are discussing, such
> as for first order PA.
>
> You can add SYMBOLS to extend a language, but the definition of
> 'formula' (of which the set of sentences is then defined as a certain
> proper subset) is INDUCTIVE so once you add a symbol you add NOT just
> one sentence. The notion of adding just a single sentence to a
> language is ludicrous.
>
> > If you do you would have gotten an inconsistency.
>
> We don't add JUST a SINGLE SENTENCE to a language, as I explained
> above.

> And that is what Godel indirectly did. He managed to add a sentence of the form P&~P−>Q to a first−order language (via an indirect transalation procedure) but did not change the logic or its principles of proof.

No, he didn't. We can add symbols to a language, thus extending a language to a new language, either conservatively (through definitions) or non−conservatively. But we don't add just a SENTENCE to LANGUAGE. If you think Godel did, then please be specific.

>> So that I can communicate at all with you, please tell me
>> what basic
>> textbooks in mathematical logic you ordinarily consult.
>> Because a
>> notion such as adding just a sentence to a language is so
>> whack that I
>> need to be able to refer to the appropriate sections of a
>> textbook to
>> get your straightened out about such basics before we can
>> productively
>> continue.

> I said if you did that, you would get an inconsistency.

Nevermind CONSISTENCY here. This has to do with LANGUAGES even ASIDE from theories. You CAN'T add just a sentence to a LANGUAGE. Get it through your head: The set of formulas for a language is defined INDUCTIVELY, so when we add a SYMBOL we get not just one sentence added.

> What I was
> trying to convey to you is that such a thing is not possible without changing the logic.

It's not POSSIBLE AT ALL.

> But Godel did that through the back−door.

Godel didn't use any "doors" − back, trap, Dutch or otherwise − to add just a sentence to a language.

If
P&~P−>Q has been translated to a formula S
in the first−order language
of PA, and if PA proves S, are you denying
that PA has *effectively*
proven itself consistent?

OF COURSE I deny that! It's ridiculous! A theory doesn't
prove itself
consistent just for proving a particular TAUTOLOGY!

Read what I have written above. You dont even begin to understand what
I am talking about.

You asked me the question (where S is a certain tautology) "if PA
proves S, are you denying that PA has *effectively* proven itself
consistent?"

If I understand that question in ordinary English, then it's a STUPID
question.

A theory does not prove itself consistent just by proving some
tautology!

A first−order theory does not prove *the*
tautology, P&~P−>Q. There is no such thing as "THE" tautology.

No, THE tautology S that you mentioned!

Again, do NOT read a book on logic. You have missed the forest for the
trees. Here it is again. NO first−order theory proves any sentence of
the form P&~P−>Q or ~(P&~P), etc. because these are sentences with
variables in them.

I am the person here who first mentioned that. Why are you repeating
over and over to lecture me on a disctinction that *I* was so forceful
to make in the first place and have adhered to all along?!

You are right when you say that "There is no
tautology in the language of a theory that is not a theorem of the
theory". Because all tautologies are theorem schemes, not theorems,

> WRONG WRONG WRONG. A tautology is a formula that is tautologically valid. And every theory has as theorems all sentences of the language that are tautologies.
>
> > and are metamathematically true,
>
> They're LOGICALLY true, thus, a fortiori, mathematically true.

As far as I am concerned, anything that is asserted as true but which is not provable in first order theories is metamathematically true.
The point I am making is very simple. Consider the theory PA and let us take (P&~P)−>Q, for a fixed P. The assertion that every sentence in the language of PA (as represented by the variable Q) follows from the contradiction P&~P is not provable in PA. While all sentences represented by this schema are tautologies, PA does not recognize the notion of "all sentences" and so cannot prove that "All sentences follow from a contradiction". So as far as I am concerned, this is a metamathematical (or metatheoretical if you like) truth *about* PA, but not something that is provable *in* PA. Do you agree?

> > in the sense that first−order
> > theories cannot recognize the existence of a schema (or sentential variables). Again: First−order theories can only prove all instances of tautologies
>
> No, the theory proves all TAUTOLOGIES in the language of the theory, thus all TAUTOLOGIES that are instances of a schema of tautologies of the language.
>
> Meanwhile, you've made no point about anything in this exchange. And in this exchange you've shown no "forest" that your great vision has seen but not seen by me or by anyone else half way familiar with this subject.

The point I have been making all along has gotten lost in the wrangling. It is the following. Post Godel, we find that PA does, after all, *prove* that "All sentences follow from a contradiction P&~P", contrary to what I have asserted in the previous paragraph. See the following thread in which Aatu pointed this out to me and my objection to the same:

http://groups.google.la/group/sci.logic/msg/cc1af3d1f93ca3f1

I am asserting that no first−order theory using only finitary principles, like PRA or PA, can ever prove something about "all sentences". This is a fundamental limitation at first order and we set up first order logic accepting this as a limitation. Having done that, we cannot contradict ourselves and accept that PA does after all, prove that all sentences follow from a contradiction.

Godel apparently managed to translate the entire set of tautologies represented by the schema P&~P−>Q into a single sentence S in the language of PA. Obviously S must quantify over sentences (indirectly, via coding) and is provable in PA. That is the only way in which PA can prove that "All sentences follow from a contradiction". I am asserting that PA should not prove S because we already agreed that PA cannot prove that "All sentences follow from a contradiction" while formlating first order logic..PA cannot refute S either. So S ought to be undecidable in PA, but such undecidability would result in a contradiciton. The only way out is to deny the validity of the translation.

I am also asserting that the schema P&~P−>Q essentially expresses that a "A proof of a contradiction is impossible", and to negate this schema, PA has to prove *some* contradition (any one will do). So S is essentially equivalent to a consistency sentence for PA and ought not to be provable in PA even if we accept Godel.

Regards, RS
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