

# Re: Godel's comments about the "true reason" for incompleteness

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- *From:* "R. Srinivasan" <[sradhakr@xxxxxxxxxxx](mailto:sradhakr@xxxxxxxxxxx)>
  - *Date:* Fri, 21 Mar 2008 15:46:47 -0700 (PDT)
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On Mar 21, 11:42 pm, MoeBlee <[jazzm...@xxxxxxxxxxx](mailto:jazzm...@xxxxxxxxxxx)> wrote:

On Mar 21, 2:53 am, "R. Srinivasan" <[sradh...@xxxxxxxxxxx](mailto:sradh...@xxxxxxxxxxx)> wrote:

On Mar 21, 3:45 am, MoeBlee <[jazzm...@xxxxxxxxxxx](mailto:jazzm...@xxxxxxxxxxx)> wrote:> Re: Mar 20, 10:55 am, "R. Srinivasan" <[sradh...@xxxxxxxxxxx](mailto:sradh...@xxxxxxxxxxx)>:

After having responded to that post, I think it would help to focus on just, say, two of the most obviously ridiculous things in it:

Now think of the following situation. Let us add  $P \& \sim P \rightarrow Q$  (and only this single sentence, where P and Q are arbitrary) into the language of PA,

Please get back after you've consulted a logic book to see that it makes no sense to say that we add just a single sentence to a language.

Of course I said "inconsistency will follow".

NO, NOTHING will follow, because it CAN'T HAPPEN. There is no action of adding just a sentence to a language. The set of sentences of a language is arrived at by INDUCTION (either first to get the set of formulas, or, I guess, you could do the induction straightaway to get

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the set of sentences). You can't just intrude arbitrarily into an INDUCTION to throw in just a single sentence.

Why do you make such a big deal of this?

Because you're using a completely LUDICROUS notion as part of your argument.

I tried to make the point that what Godel did amounts to precisely what you are ridiculing.

Then SHOW it. Please give the exact passage in his papers where you think he does that.

Again: No first-order theory ever proves a tautology. That is your misunderstanding.

No, YOUR misunderstanding. Or at least it is your misunderstanding to overlook that there are quite ordinary definitions of 'tautology' that yield that every first order theory proves EVERY tautology in the language of that theory.

There is no such thing as \*a\* tautology as far as first-order languages are concerned.

WRONG. We have a perfectly rigorous definition of 'tautology' for first order languages.

And such rigorous definitions only reflect the common notion that a tautology is a sentence that is true under all truth table evaluations of its sentential components. Under certain common definitions, what is true or not is the SENTENCE and NOT the SCHEMA of sentences. All INSTANCES of a schema have truth values, but the schema ITSELF does not have a truth value. The schema itself is a formula that defines a SET of sentences. It's not a SET of sentences that has a truth value, but rather the sentences themselves.

Fine. I have no problem with this definition of tautology. The point is that a theory like PA cannot say something about a SET of sentences, because quantification over sentences is not permissible at first order. So PA cannot prove that "Every sentence follows from a

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contradiction". That is all I wish to emphasize here. Secondly if you do permit quantification over P and Q, then the sentence

(For all P) (For all Q) ((P&~P) →Q)

does have truth value. Namely it is true, but such a truth must be metamathematical with respect to a first order theory like PA, since PA cannot ever say something about "all sentences". I hope you accept this as a fundamental limitation of first-order logic. If you do, please look at my reply to another post in which I raise the issue of how Godel apparently managed to overcome this fundamental limitation of first-order logic, using only finitary reasoning within first-order logic (as claimed by Godelians; I do not agree with the "finitary" bit). How on earth is this possible? Do you have any explanation?

First order languages can only recognize instances of a schema representing a tautology.

The instances ARE tautologies. Moreover, though we may use schemata for certain purposes, the definition of 'tautology' for first order languages does not go through schemata.

The tautology itself, say,  $P \& \sim P \rightarrow Q$  where P and Q are variables,

Not under certain ordinary definitions of 'tautology'. Under certain ordinary definitions, if 'P' and 'Q' are meta-variables ranging over formulas or over sentences, as the case may be, then ' $P \& \sim P \rightarrow Q$ ' is not ITSELF a tautology but rather it is a schema such that all its INSTANCES are tautologies. However, of course, we're not always so pedantic about that distinction so that we refer to ' $P \& \sim P \rightarrow Q$ ' also as a tautology even though, if we were precise, we would say it's a schema such that all its INSTANCES are tautologies. (And I don't preclude that there might be authors who define 'tautology' so that a schema itself is a tautology, but rather I'm saying that given certain common definitions of 'tautology', under those definitions, the schema is not itself a tautology but rather its instances are.)

I do not have any essential disagreement over what you have said above. Thanks for the clarification.

Regards, RS

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