Re: Z−Infinity

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- *From*: Zaljohar@xxxxxxxxx
- *Date*: Sun, 13 Apr 2008 09:30:19 −0700 (PDT)

On Apr 10, 2:15 pm, lwal...@xxxxxxxxx wrote:

> On Apr 8, 5:11 am, Zaljo...@xxxxxxxxx wrote:
>
> > Size Theory is set of all sentences entailed (from first order logic with identity '=' and epsilon membership 'e' and the primitive constant Z , and the primitve two place relation symbole '< ' to denote 'smaller than' ,and the primitive one place function symbole 'S' to denote 'size') by the non
> > logical axioms of ZF and the following non logical axioms:
>
> I've been thinking of a similar theory myself for a while. I wonder whether this theory can be consistent.
>
> My pet theory is to be an extension of ZF−Infinity. But instead of an infinite set omega, I have a new type of infinite set, which I call "alpha" (the first Greek letter, as omega is the last), which is similar in many ways to Zuhair's Z.
>
> Rather than attempt to write any new axioms, let me simply state the properties that alpha is supposed to have.
>
> −− Alpha is an "ordinal," or at least resembles an ordinal in that
> it's
> a transitive set, all of whose elements are themselves transitive.
> −− Alpha is in some ways "infinite," but in other ways "finite." In particular, it's D−finite, the weakest type of finite.
> −− Alpha contains as elements 0, 1, 2, 3, etc.
> −− Alpha contains what appears to be a infinitely descending e−chain, alpha_1ealpha, alpha_2ealpha_1, alpha_3ealpha_2, etc.
>
> Now you're probably thinking, hold on a minute! This already seems very inconsistent! I said ZF−Infinity, so this includes the Axiom of Foundation/Regularity, which forbids infinitely descending chains, doesn't it?
>
> Well, the Axiom of Regularity literally states that every set must be

disjoint with one of its elements. And since 0ealpha, 0 is the
element of alpha with which is distinct.

If you think about it, Regularity alone doesn't forbid the existence
of
an infinite chain per se! The proof of the nonexistence of an infinite
chain actually requires the Axiom of Infinity, by starting with omega
and applying the Replacement Schema to produce a set

{alpha, alpha_1, alpha_2, alpha_3, ...}

and it's this set that fails to be disjoint with any of its elements!
But
since I (conveniently) left out the Axiom of Infinity, we can't
conclude
that this is an ill