Re: Minimal logic valid?

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translogi schrieb:

> The main difference is that natural deduction uses lots of inference
> rules and no axioms.
> while the axiomatic method uses axioms(chemata) and only detachment /
> modus ponens

Hilbert style calculi usually have no detachment rule.
Natural deduction has it. What are you talking
about?

In natural deduction you have a context G, and
you can formulate a rule as such:

G |− A
−−−−−−−−−−
G\B |− B−>A

Or graphically:

[B]
A
−−−−−
B −> A

Hilbert style calculi dont have this context.
They have just rules over a single sukzedent
sequents with zero context.

But a hilbert style calculi might make use
of axioms G, and then we write also:

G |− A

And we might have the meta result, that when
G |− A is valid, that G\B |− B−>A is also
valid. That is called the deduction theorem.

The deduction theorem states that when there
is a hilbert style proof G |− A, then
there is "another" hilbert style proof of
G\B |− B−>A.

That hilbert style methods need more axiom schemas,
I agree. Namely to be classical, because things
go rather into the axioms than into the rules.

But that hilbert style is an axiomatic method,
while natural deduction is not, I wouldn't
agree. For example as soon as one has the
classical apparatus in both system, one can
work the same way with axiom systems.

Also an axiom schema is a degenerated instance
of the following pattern of a rule:

Template_1 .... Template_n
−−−−−−−−−−−−−−−−−−−−−−−−
Template

An axiom schema has just n=0, i.e. no precluding
patterns. So we can compare hilbert style and
natural deductions:

System Templateform n
−−−
Hilbert style |− A =0 for the logic axioms
=0 for the axioms
=2 for MP
Natural deduction G |− A =1, 2, 3 for the logic rules
=0 for axioms and assumptions
=1 for detachment
=2 for MP

I think both systems belong to the axiomatic method
as there are some principles (the rules with n=0
or n<>0) layed down, and we try to work from these
principles.

Bye
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