Re: Godel cant tell us what makes a mathematical statement true

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Source: http://sci.tech−archive.net/Archive/sci.logic/2008−08/msg00195.html

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- *From*: herbzet <herbzet@xxxxxxxxx>
- *Date*: Mon, 11 Aug 2008 04:28:26 −0400

Nam Nguyen wrote:

> herbzet wrote:
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>> Nam Nguyen wrote:
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>>> Baudouin Le Charlier wrote:
>>>
>>>> A statement is true just because it is true not
>>>> because somebody
>>>> 'relies on the notion of truth'.
>>>
>>> Could you think of any circumstance in which 0=1 is true?
>>
>> Sure. Let "0" be my dad. Let "1" be me. Let "=" be the
>> relation "father of".
>
> Can't you think a bit more mathematical,

Well, I do what I can.

> like:
>
> A1: AxEy[Sx=y]
> A2: Ax[0=x]
>
> Then (as a theorem) 0=S0, which is 0=1. Right?

Right.

> Not sure where you're going with this.

> Now T df= {A1 + A2} is consistent, so 0=1 is *true* in [any
> model of] T.

Right. It's false in some structures that are not models of T.

> So, unlike what Baudouin said, whether 0=1, or 0=/=1,
> is true does rely on some notion of truth!

What you've shown is that whether 0=1 (or 0=/=1) is true relies
on the structure in which it is interpreted. This, in itself,
relies on a model theoretic notion of truth.

> There's no such thing
> as a formula being true "just because it is true", and not relying
> on a notion of truth.

Right. Truth attaches to statements, not to signs such as
formulae. A sign must be interpreted to express a statement.
Whether an interpreted sign, a statement, expresses a truth
or not will rely on what we take as the relationship between
statements and truth, i.e. upon some notion of truth.

The model theoretic notion of truth is that an interpreted
formula is true in a structure when the formula is satisfied
by that structure. This is essentialy a rigorised formulation
of Aristotle's correspondence theory of truth, in which a
statement is true when it corresponds to reality.

A sympathetic reading of M. Le Charlier's assertion would be
that the correspondence of a statement to reality, or the
satisfaction of an interpreted formula in a structure, is
not a matter of opinion, or of proof, or of our knowledge
of such correspondence or satisfaction.

Of course you are right: to assert of any statement that
it "is true" or "is false" is to already have some notion
of what it means for a statement to "be true" or to "be false".

> > To think that something is true only because
> > it has been proven is the wrongest idea you
> > can conceive.

> > > Suppose for a given formal system T, we define true
> > > sentences
> > > as the following:
> > >
> > > − T(F) = true iff T |− F.
> > > − T(F) = false otherwise.
> > >
> > > What would you think as "wrong" with this definition?

> > Well, if T |/− F and T |/− ~F then both F and ~F are false.
> > Is that problematic for you?

> Of course not. It's *not a perfect definition* but:
>
> a) If a formula is not defined to be true w.r.t. T, then there's
> nothing wrong to equate it with being false. In this way then:
>
> − A consistent system is one in which one of {F, ~F} is true and
> the other false.
> − An inconsistent system is one in which both F and ~F are true.
> − F is undecidable in T if both F and ~F are false (which only means
> F and ~F aren't provable in T!)

If both F and ~F are false (which only means F and ~F aren't provable
in T) then F is undecidable in T (by your third clause) and T is
not a consistent system (by your first clause), since it will not
be the case that one of {F, ~F} is true (provable) and the other
false (not provable).

So we have that a system that is is not consistent is not necessarily
the same as a system that is inconsistent (by your second clause):
F and ~F are both true (provable).

I don't find this terminology quite satisfactory.

Also, I think you have to think about the difference between the
concept of a formula F being true in a theory T (being true in
every model of T) and the the anterior notion of a formula F being
true in a given model (structure).

> Nothing would seem "wrong" at all!
>
> b) The standard definition of truth is *not perfect either*: there
> are some T in which if T is consistent, there would be some formula
> F which you can't tell whether or not F is true in any model
> of T!

You seem to wish to assert that the truth of a formula F relies
on our knowledge (whether we can "tell") of its truth in some model.

Commonly (but not universally) it is taken that the truth or
falsehood of a statement is not dependent on its epistemological
status of being known to be true or not. This is part of the
realist (Platonist) position that there is an objective reality
that is not dependent on our minds, or what we happen to know.

Whether or to what degree mathematical reality partakes of this
objective status is, of course, a venerable debate.

(You have provoked in me some thought about how far the model
theoretic notion of truth is implicitly a realist position,
or not.)

> But the point here is a formula can't be just simply true. You have to
> *choose* *some* selected/defined notion of truth!

True. But once having chosen some notion of what it means
for a statement to "be true" or to "be false", then it
is no longer a matter of opinion as to whether a statement
is true or false −− it is only a question of whether the
statement is in accord with the notion adopted.

Of course, some notions of what "truth" is will be sillier than others.

You seem to have two issues here: that a formula must be interpreted
to have any meaning at all (much less be true or false), and that
a formula has to be known to be true (by some criteria of truth) in
order for it to be true (by that criteria).

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hz
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