Re: Godel cant tell us what makes a mathematical statement true

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- *From*: Nam Nguyen <namducnguyen@xxxxxxx>
- *Date*: Tue, 12 Aug 2008 22:28:31 GMT

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herbzet wrote:

> Nam Nguyen wrote:
>
>> With the revised above, I hope you'd be convinced to change your mind.
>
> It's pretty standard as amended, other than the fact that you
> are identifying truth with provability.

Arguably, there are "forces" within the community who'd tend to
oppose such a truth definition I've given. I think in their opinion
that would reduce their sacred arithmetical truths to just syntactical
provabilities within Q; and as such, Godel's results would be groundless.

> Still, we have the situation that if T |/− F and T |/− ~F then
> F and ~F are both false.

I assume the "situation" you alluded to below.

> This doesn't distinguish refuted
> formulae (the negations of proven formulae) from undecidable
> formulae.

But it does. According to the definition, if F is a refuted formula
then ~F is provable (criteria 1). On the other hand, if F is undecidable
then both F *and* ~F aren't provable (i.e. false)! There's nothing
to be confused about and nothing technically wrong here. In other
words such a definition is *just an alias* for the syntactical definitions
of provability, reputability, (un)decidability, (in)consistency!

> > > Also, I think you have to think about the difference between
> > > the
> > > concept of a formula F being true in a theory T (being true in
> > > every model of T) and the the anterior notion of a formula F
> > > being
> > > true in a given model (structure).
> > >
> > > > Nothing would seem "wrong" at all!
> > > >
> > > > b) The standard definition of truth is *not
> > > > perfect either*: there
> > > > are some T in which if T is consistent, there
> > > > would be some formula
> > > > F which you can't tell whether or not F is
> > > > true in any model
> > > > of T!
>
> In the standard definition of truth, a formula F that is undecidable
> in a theory T is true in some model of T. If F is false in every
> model of T, then F is decidable in T: it is the negation of a
> formula provable in T (F is refuted in T). I don't see that
> as an imperfection.

The imperfection lies in that for complex truth systems, such as arithmetic
truth, the canonical definition (which is intuitive) would be treated
as *another entirely independent regime of reason*, independent from
syntactical provability through inference rules. And that's not only
"dangerous" to reasoning it's also wrong as a framework that we could
rely on − to obtain *new knowledge*.

> > > You seem to wish to assert that the truth of a formula F relies
> > > on our knowledge (whether we can "tell") of its truth in some
> > > model.
> >
> > That's what I'd like to convey. The truth of a formula F is relative
> > to whatever definition of truth one is please. And in my case, I've
> > "relativized" it to that of syntactical proof.
>
> What if a formula F has a proof but you don't know that it has
> a proof?

What about it? Are you saying that one human being must know all
1st order proofs there are?

> Is F false until you discover the proof? Or was it
> true all along?

According to the definition, F is true when T |− F, or false if
it's not true. Period. Whether or not one discovers the proof or
the un−proof is immaterial to its intrinsic truth value (again, per
this definition).

You sounded as if "F's being provable" and "one's not knowing its
proof" must be contradictory! They're not, unfortunately. That's
actually a legacy of FOL (a.k.a. the "Characterization Problem"
for F)!

> I didn't say "formula", I said "statement".

In the context we're discussing, what's the significant difference
between a formula and a statement? (We're not talking about meta
statements here, are we?) To many "zero is equal to one" is a statement,
but that could be converted to the formula "0=S0", given the context. No?

> Of course a formula
> can assume different truth values depending on the structure in
> which you interpret it. This doesn't mean that you automatically
> know its truth value in that structure.

I didn't say anything about "automatically", did I? I'm not sure I follow
your argument here.

>> Whether or to what degree mathematical reality partakes of
>> this
>> objective status is, of course, a venerable debate.
>>
>> (You have provoked in me some thought about how far the
>> model
>> theoretic notion of truth is implicitly a realist position,
>> or not.)

All I know is if mathematical reasoning is about (or dependent on) knowledge, then the reasoning can't go too far: either by means of provability − or truth!

I don't know what this means.

It means basically that, e.g., if you have a problem of *not* knowing a syntactical proof for (F /\ ~F), say, in Q, then *assuming* there's a model of Q is just that: an *assumption*, *not* a fact!

But the point here is a formula can't be just simply true. You have to *choose* *some* selected/defined notion of truth!

True. But once having chosen some notion of what it means for a statement to "be true" or to "be false", then it is no longer a matter of opinion as to whether a statement is true or false −− it is only a question of whether the statement is in accord with the notion adopted.

Of course, some notions of what "truth" is will be sillier than others.

You seem to have two issues here: that a formula must be interpreted
to have any meaning at all (much less be true or false),

No! Formula's semantic and truth don't have to be identical.

That's not what I said. An uninterpreted formula has no meaning or truth value. Statements have truth values.

There's no such thing as just a true formula: usually it's understood by that we mean an interpreted−as−true formula. As such an interpreted formula needs to have a meaning. But not the other way around. For example, if the formula F = (0=0) /\ (0=1) has a meaning, its meaning is *independent* from whether or not F has any truth value!

For example,
GC has some meaning but whether or not it's true or false (by whatever the chosen underlying truth definition) is a different matter.

> Well maybe you have a point here: Assume the standard semantics
> for arithmetic, that is, assume, e.g., that Euclid's theorem
> on the infinitude of prime numbers means what we usually take
> it to mean. How do you vary its truth−value within the usual
> semantics? I can weaken PA to where it can't prove Euclid's
> theorem, but the statement of the theorem will still be true
> in the standard model of PA, no?

> > > and that a formula has to be known to be true (by some
> > > criteria of truth)
> > > in order for it to be true (by that criteria).

The above is what you stipulated as one of my issue.

> > That's I think is meant by something like "mathematical truth is relative",
> > and is what I'd like to convey. But that in itself is not an "issue", imho.

A slight mis−communication here, imho. (Though I did say "like").
Had your statement above been "a formula has to be known to be
true (by some criteria of truth) in order for it to be *asserted as*
true (by that criteria)", then that wouldn't be an issue (mine or otherwise),
and I'd agree, in saying "mathematical truth is relative".

But I now realize that's not what you really said. So...

> You wish to say that mathematical truth of a statement is
> relative not only to the chosen criteria of truth, but
> also to whether we know the statement meets the criteria?

that's not exactly what I wish to convey: "not only" isn't part
of what I intended to say!

> IOW, a statement is false if it fails to meet the chosen
> truth definition, but also if we don't know whether it meets
> the truth definition or not?

A formula is false when "it fails to meet the chosen truth definition",
whether or not we know about that is another issue all together.

−−
"To discover the proper approach to mathematical logic,

we must therefore examine the methods of the mathematician."
(Shoenfield, "Mathematical Logic")
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