

Re: Godel cant tell us what makes a mathematical statement true

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- *From:* Nam Nguyen <namducnguyen@xxxxxxx>
 - *Date:* Fri, 15 Aug 2008 01:49:38 GMT
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MoeBlee wrote:

On Aug 12, 3:28 pm, Nam Nguyen <namducngu...@xxxxxxx> wrote:

herbzet wrote:

Nam Nguyen wrote:

With the revised above, I hope you'd be convinced to change your mind.

It's pretty standard as amended, other than the fact that you are identifying truth with provability.

Arguably, there are "forces" within the community who'd tend to oppose such a truth definition I've given. I think in their opinion that would reduce their sacred arithmetical truths to just syntactical provabilities within Q; and as such, Godel's results would be groundless.

Suppose we define 'true' as 'provable' as you suggest. Then we can define 'troo' as we had previously defined 'true'.

What do you mean by "Then" here? Anybody can define 'true' in any which way and create an alias ('troo', 'truthful', ...) *at will*, so "Then" isn't necessary. The point I made here is "provable" is a well established syntactical notion in FOL; so if we use it to define 'true', as opposed to the canonical definition of 'true' which is based on intuition, then we have to accept certain consequences. And I've mentioned these consequences in the above sentence "I think ... Godel's results would be groundless."

Now we're right back to the usual results in mathematical logic (including variations on the incompleteness theorem that mention 'true') except 'troo' occurs where 'true' used to appear.

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No! Not if Incompleteness results we're talking about. You misunderstood the issue. Here, 'troo' is an alias for the old notion of 'true', while my 'true' is an alias for 'provable', but the old 'true' and the new 'true' *aren't* of the same sense. And whether we'd arrive at the same Godel's results would depend *which definition* of 'true'/'false' we're using.

Still, we have the situation that if $T \not\vdash F$ and $T \not\vdash \sim F$ then F and $\sim F$ are both false.

I assume the "situation" you alluded to below.

This doesn't distinguish refuted formulae (the negations of proven formulae) from undecidable formulae.

But it does. According to the definition, if F is a refuted formula then $\sim F$ is provable (criteria 1). On the other hand, if F is undecidable then both F *and* $\sim F$ aren't provable (i.e. false)! There's nothing to be confused about and nothing technically wrong here. In other words such a definition is *just an alias* for the syntactical definitions of provability, reputability, (un)decidability, (in)consistency!

But why would we WANT an alias? We have clear definitions of those those things.

The aliases are here only to demonstrate to the readers that, for *better* reasoning, we should base mathematical assertions on syntactical provability rather than the old intuitive notion of "truth". For example, to prove F is undecidable, with 100% certainty, we should prove so using *only* syntactical rules of inferences and axioms!

Why can't we have a different definition of 'true', I mean 'troo'?

We'd want to have a different and new definition of 'true' because the old one isn't adequate. But your "Why can't we have a different definition of 'troo'?" wouldn't quite make sense, because there isn't such a thing as the old definition of 'troo'!

And doesn't "Both F and $\sim F$ are both false" seem at least a bit odd to you as far as an ordinary non-technical meaning of 'false'?

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No. We're supposed to live in a binary (logic) world aren't we?
If an F isn't provable then got to be non-provable. So, in this sense,
if F isn't true then it got to be false. Nothing is odd, it seems to me!

And I think what herbzet meant is that, with your approach, both a
refuted formula and an undecidable formula both come out as false,
which does seem at least counterintuitive.

Also, I think you have to
think about the difference
between the
concept of a formula F
being true in a theory T
(being true in
every model of T) and the
the anterior notion of a
formula F being
true in a given model
(structure).

Nothing
would seem
"wrong" at
all!
b) The
standard
definition of
truth is *not
perfect
either*:
there
are some T
in which if
T is
consistent,
there would
be some
formula
F which you
can't tell
whether or
not F is true
in any
model
of T!

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In the standard definition of truth, a formula F that is undecidable in a theory T is true in some model of T . If F is false in every model of T , then F is decidable in T : it is the negation of a formula provable in T (F is refuted in T). I don't see that as an imperfection.

The imperfection lies in that for complex truth systems, such as arithmetic truth, the canonical definition (which is intuitive) would be treated as *another entirely independent regime of reason*, independent from syntactical provability through inference rules. And that's not only "dangerous" to reasoning it's also wrong as a framework that we could *rely* on – to obtain *new knowledge*.

But we can formalize the mathematical definition of 'truth' so that we can formally prove that certain sentences are true or false in certain models.

Any of us can formalize a particular of 'truth' and use it to "prove" something else. That's not the problem! The problem is if such truth definition has anything to do with a particular formal system T , then how well would such definition would rhyme with the consistency of T ?

I believe my definition does very well but the canonical doesn't!

Of course, we don't have an algorithm to determine such questions; but we don't have an algorithm to determine questions of plain first order provability either.

Correct. But that's why if we assume the natural is a model of Q then we have to admit that's just an assumption and *not a proof*. Consequently, Godel's results are hypothetical results, not assertions! For them to be assertions, the required additional hypothesis would be:

"If Q is syntactically consistent"

Godel didn't stipulate that one hypothesis. And neither have we, after 70+ years!

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"To discover the proper approach to mathematical logic, we must therefore examine the methods of the mathematician."
(Shoenfield, "Mathematical Logic")

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