Re: Godel cant tell us what makes a mathematical statement true

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- *From*: herbzet <herbzet@xxxxxxxxx>
- *Date*: Fri, 15 Aug 2008 00:54:24 −0400

Nam Nguyen wrote:

> MoeBlee wrote:
>
> > On Aug 12, 3:28 pm, Nam Nguyen <namducngu...@xxxxxxx> wrote:
> >
> > > herbzet wrote:
> > >
> > > > Nam Nguyen wrote:
> > > >
> > > > > With the revised above, I
> > > > > hope you'd be convinced to
> > > > > change your mind.
> > > >
> > > > It's pretty standard as amended, other than
> > > > the fact that you
> > > > are identifying truth with provability.
> > >
> > > Arguably, there are "forces" within the community who'd
> > > tend to
> > > oppose such a truth definition I've given. I think in their
> > > opinion
> > > that would reduce their sacred arithmetical truths to just
> > > syntactical
> > > provabilities within Q; and as such, Godel's results would be
> > > groundless.
> >
> > Suppose we define 'true' as 'provable' as you suggest. Then we can
> > define 'troo' as we had previously defined 'true'.
>
> What do you mean by "Then" here? Anybody can define 'true' in any which
> way and create an alias ('troo', 'truthful', ...) *at will*, so "Then"
> isn't necessary. The point I made here is "provable" is a well established

syntactical notion in FOL; so if we use it to define 'true', as opposed to
the canonical definition of 'true' which is based on intuition, then we have
to accept certain consequences. And I've mentioned these consequences in the
above sentence "I think ... Godel's results would be groundless."

> Now we're right
> back to the usual results in mathematical logic (including variations
> on the incompleteness theorem that mention 'true') except 'troo'
> occurs where 'true' used to appear.

No! Not if Incompleteness results we're talking about. You misunderstood
the issue. Here, 'troo' is an alias for the old notion of 'true', while my
'true' is an alias for 'provable', but the old 'true' and the new 'true'
aren't of the same sense. And whether we'd arrive at the same Godel's
results would depend *which definition* of 'true'/'false' we're using.

> > > Still, we have the situation that if T |/− F and
> > > T |/− ~F then
> > > F and ~F are both false.
> >
> > I assume the "situation" you alluded to below.
> >
> > > This doesn't distinguish refuted
> > > formulae (the negations of proven formulae)
> > > from undecidable
> > > formulae.
> >
> > But it does. According to the definition, if F is a refuted
> > formula
> > then ~F is provable (criteria 1). On the other hand, if F is
> > undecidable
> > then both F *and* ~F aren't provable (i.e. false)! There's
> > nothing
> > to be confused about and nothing technically wrong here. In
> > other
> > words such a definition is *just an alias* for the syntactical
> > definitions
> > of provability, reputability, (un)decidability, (in)consistency!
>
> But why would we WANT an alias? We have clear definitions of those
> those things.

The aliases are here only to demonstrate to the readers that,
for *better* reasoning, we should base mathematical assertions
on syntactical provability rather than the old intuitive notion

of "truth". For example, to prove F is undecidable, with 100% certainty, we should prove so using *only* syntactical rules of inferences and axioms!

If a theory T proves a sentence is undecidable in T, then T is inconsistent. (Not actually sure this is correct ("true") in complete generality.)

Why can't we have a different definition of 'true', I mean 'troo'?

We'd want to have a different and new definition of 'true' because the old one isn't adequate. But your "Why can't we have a different definition of 'troo'?" wouldn't quite make sense, because there isn't such a thing as the old definition of 'troo'!

And doesn't "Both F and ~F are both false" seem at least a bit odd to you as far as an ordinary non−technical meaning of 'false'?

No. We're supposed to live in a binary (logic) world aren't we?
If an F isn't provable then got to be non−provable. So, in this sense,
if F isn't true then it got to be false. Nothing is odd, it seems to me!

And I think what herbzet meant is that, with your approach, both a refuted formula and an undecidable formula both come out as false, which does seem at least counterintuitive.

Also, I think you have to think about the difference between the concept of a formula F being true in a theory T (being true in every model of T) and the the anterior notion of a formula F being true in a given model (structure).

Nothing would seem "wrong" at all! b) The standard definition of truth is *not perfect either*: there are some T in which if T is consistent, there would be some formula F which you can't tell whether or not F is true in any

model
of
T!

In the standard definition of truth, a formula
F that is undecidable
in a theory T is true in some model of T. If F
is false in every
model of T, then F is decidable in T: it is the
negation of a
formula provable in T (F is refuted in T). I
don't see that
as an imperfection.

The imperfection lies in that for complex truth systems, such
as arithmetic
truth, the canonical definition (which is intuitive) would be
treated
as *another entirely independent regime of reason*,
independent from
syntactical provability through inference rules. And that's not
only
"dangerous" to reasoning it's also wrong as a framework that
we could
rely on − to obtain *new knowledge*.

But we can formalize the mathematical defintion of 'truth' so that we can formally prove that certain sentences are true or false in certain models.

Any of us can formalize a particular of 'truth' and use it to "prove"
something else. That's not the problem! The problem is if such truth
definition has anything to do with a particular formal system T, then
how well would such definition would ryhme with the consistency of T?

I believe my definition does very well but the canonical doesn't!

Of course, we don't have an algorithm to determine such questions; but we don't have an algorithm to determine questions of plain first order provability either.

Correct. But that's why if we assume the natural is a model of Q then
we have to admit that's just an assumption and *not a proof*.
Consequently, Godel's results are hypothetical results, not assertions!
For them to be assertions, the required additional hypothesis would be:

"If Q is syntactically consistent"

Godel didn't stipulate that one hypothesis. And neither have we, after
70+ years!

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"To discover the proper approach to mathematical logic,
we must therefore examine the methods of the mathematician."
(Shoenfield, "Mathematical Logic")

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