Re: Godel cant tell us what makes a mathematical statement true

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- *From*: herbzet <herbzet@xxxxxxxxx>
- *Date*: Fri, 15 Aug 2008 00:53:40 −0400

MoeBlee wrote:

> Nam Nguyen wrote:
>
>> herbzet wrote:
>>
>>> Nam Nguyen wrote:
>>>
>>>> With the revised above, I hope you'd be
>>>> convinced to change your mind.
>>>
>>> It's pretty standard as amended, other than the fact that you
>>> are identifying truth with provability.
>>
>> Arguably, there are "forces" within the community who'd tend to
>> oppose such a truth definition I've given. I think in their opinion
>> that would reduce their sacred arithmetical truths to just syntactical
>> provabilities within Q; and as such, Godel's results would be groundless.
>
> Suppose we define 'true' as 'provable' as you suggest. Then we can
> define 'troo' as we had previously defined 'true'. Now we're right
> back to the usual results in mathematical logic (including variations
> on the incompleteness theorem that mention 'true') except 'troo'
> occurs where 'true' used to appear.
>
>>> Still, we have the situation that if T |/− F and T |/− ~F then
>>> F and ~F are both false.
>>
>> I assume the "situation" you alluded to below.

> > > This doesn't distinguish refuted
> > > formulae (the negations of proven formulae) from
> > > undecidable
> > > formulae.

> > But it does. According to the definition, if F is a refuted formula
> > then ~F is provable (criteria 1). On the other hand, if F is undecidable
> > then both F *and* ~F aren't provable (i.e. false)! There's nothing
> > to be confused about and nothing technically wrong here. In other
> > words such a definition is *just an alias* for the syntactical definitions
> > of provability, refutability, (un)decidability, (in)consistency!

> But why would we WANT an alias? We have clear definitions of those
> those things. Why can't we have a different definition of 'true', I
> mean 'troo'?

To argue Nam's position for a moment: It's not an alias, it a *clarification*. Nam says, that's what "truth" means −− provability.

Or maybe not. Maybe he wishes to imply that "truth" is inherently stipulative, nothing more −− a free choice.

Just btw, I have some sympathy for the mathematical−truth−is−provability position −− it was my naive concept, and I haven't given up all attachment to it.

> And doesn't "Both F and ~F are both false" seem at least a bit odd to
> you as far as an ordinary non−technical meaning of 'false'?
>
> And I think what herbzet meant is that, with your approach, both a
> refuted formula and an undecidable formula both come out as false,
> which does seem at least counterintuitive.

Yes −− saying that a formula is false in theory T doesn't tell us if it's refuted or merely undecided. Standardly, the latter means that the formula is true in some (but not all) models of T.

> > > > > Also, I think you have to
> > > > > think about the difference
> > > > > between the
> > > > > concept of a formula F
> > > > > being true in a theory T
> > > > > (being true in

every model of T) and the
the anterior notion of a
formula F being
true in a given model
(structure).

Nothing would seem "wrong" at all!

b) The standard definition of truth is *not perfect either*: there are some T in which if T is consistent, there would be some formula F which you can't tell whether or not F is true in any model of T!

In the standard definition of truth, a formula F that is undecidable
in a theory T is true in some model of T. If F is false in every model of T, then F is decidable in T: it is the negation of a formula provable in T (F is refuted in T). I don't see that as an imperfection.

The imperfection lies in that for complex truth systems, such as arithmetic truth, the canonical definition (which is intuitive) would be treated as *another entirely independent regime of reason*, independent from syntactical provability through inference rules. And that's not only

> > "dangerous" to reasoning it's also wrong as a framework that we could
> > *rely* on − to obtain *new knowledge*.

> But we can formalize the mathematical definition of 'truth' so that we
> can formally prove that certain sentences are true or false in certain
> models. Of course, we don't have an algorithm to determine such
> questions; but we don't have an algorithm to determine questions of
> plain first order provability either.

−−
hz
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