

# Re: Binary Tree and Pairs of Nodes

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On Oct 8, 12:24 pm, WM <[mueck...@xxxxxxxxxxxxxxxx](mailto:mueck...@xxxxxxxxxxxxxxxx)> wrote:

Every node in your infinite binary tree occurs at the  
END OF A FINITE path.

Since that finite path to the node IS A FINITE BIT-STRING,  
it uniquely represents A NATURAL NUMBER for the node.

Obviously, the nodes can be ordered:

the root is 1, the (node at the end of the)

terminating path 0 is 2, the terminating path 1 is 3,

the path 00 terminates at the node numbered 4, the path 01

terminates at node#5,

the path 10 terminates at the node numbered 6, and the terminating

path 11 is 7, etc.

You just order them left-to-right across each row, with 0000 on the  
left and 1111 on

the right. If you want to know the number corresponding to a path,  
just put a 1

in front of the path-as-a-bit-string and evaluate the resulting string  
as a natnum in binary.

If you want to know node n's number n, represent the path  
(from the root) to the node in binary, and replace the root with a 1  
in front (left).

Evaluate that bit-string as a natnum. That is node n's number (also  
n).

So you say you have a mapping of every path to a node.

Well, that node has a number (from above), so you also have a mapping  
from every path TO A NATURAL NUMBER.

We diagonalize this mapping by constructing the following denumerably  
long

infinite path that YOU DO NOT map TO ANY natnum:

Call your mapping  $Y(\cdot)$ .

$Y(p)=n$ , where  $p$  is some path and  $n$  is a node

in the infinite binary tree. Let the  $i$ 'th bit of path  $p$  be called

$p_i$ : if the  $i$ 'th step in the path is to the left on the binary tree,

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then  $p_i=0$ ; if it is to the right then  $p_i=1$ .

It is easy to design a path, MY path, M, that PROVABLY, for EVERY n, is NOT the path

that You map to n:  $Y(M) \neq n$  FOR ANY n.

MY PATH M IS NOT INCLUDED in your mapping, so your mapping DOES NOT include ALL the infinite paths.

First of all,  $Y(M) \neq 0$  since no path maps to 0 in this scenario.

You do not map My path to 1, because whichever path YOU mapped to 1, MY path takes THE OPPOSITE direction at step 1.

You see, your problem is, if you want to prove that there are NO MORE infinite paths

than nodes, you have to not only map each path to 1 node, you have to ALSO

map each NODE to at MOST ONE path. Otherwise you could map EVERY path to 0,

which wouldn't prove anything.

THIS MEANS YOUR MAPPING Y IS INVERTIBLE.

$Y(p)=n$  ( you falsely allege), for every path p, but if it's 1-1

and if you're not leaving out any numbers, then Y has an inverse, call it y

(it'll be clear not only from case but from type of the arguments as well,

which Y we mean)) satisfying

$y(n)=p$ . In other words, you ALSO map every NODE to some PATH.

Third of all, my path does not map to 2, because whatever path

YOU mapped to 2, MY path goes THE OTHER WAY

in its 2nd step. Do you get it yet? MY PATH IS NOT IN your map.

For all n,  $M_n=1-[y(n)]_n$ ,

where  $y(n)$  is the path that you mapped to n.

This, unlike all your handwaving about the picture,

IS A PROOF.

And it does NOT DISAGREE with your picture in ANY way (although

it does contradict it). It BEGINS BY ASSUMING THAT YOUR PICTURE IS CORRECT AND THAT YOU DO HAVE a list.

But it constructs, FROM YOUR mapping, a path that IS NOT IN YOUR mapping.

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