

Re: Cantor's "diagonal argument". My Objection.

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LudovicoVan schrieb:

On 6 Dec, 17:46, Jan Burse <janbu...@xxxxxxxxxxxx> wrote:

There are
many many(*) orders on the natural numbers.

Here I give you an order of type $\omega + \omega$ of
the natural numbers:

1 3 5 ... 2 4 6 ...

This is very interesting. A couple of questions if you don't mind:

1) I take it to be something like: 1,3,5,...|2,4,6,... How would you
define this sequence formally? (I am puzzled by the non-ending in
between);

In this case I can do it formally based
on the natural numbers. Let J denote
the ordering on $\mathbb{N} \times \mathbb{N}$. We can define it as
follows:

$J(n,m) : \Leftrightarrow$ if $\text{odd}(n) \ \& \ \text{even}(m)$ then true
else if $\text{even}(n) \ \& \ \text{odd}(m)$ then false
else $n < m$

2) What is the cardinality of this sequence (should I say, of the set
of the elements of the sequence)? I guess it is still \aleph_0 , but I
think I'd need the formal definition of the sequence to see exactly
how.

The cardinality of the domain of J is the same as
the one of the natural numbers, as the domain

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are the natural numbers.

(*) Actually an order on the natural numbers
can be coded in a real number. Guess why?

This is just as interesting: could you please elaborate a little bit?
I can't guess how that is.

We need to show that *any* J can be coded
by a real r.

First of all we code the pairs as single natural
numbers. Thus $J(n,m)$ becomes $H(\langle n,m \rangle)$, where $\langle \cdot, \cdot \rangle$
is an injective mapping from $\mathbb{N} \times \mathbb{N}$ to \mathbb{N} . There
are many such mappings available. The simplest
mapping is based on traversing the grid:

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+-----  
|0 1 3  
|2 4  
|5  
|
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$$\langle n,m \rangle = (n+m)(n+m+1)/2 +$$