Re: non−Archimedean models of Euclidean geometry?

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- *From*: Gc <Gcut667@xxxxxxxxxxx>
- *Date*: Mon, 9 Mar 2009 09:16:35 −0700 (PDT)

On 9 maalis, 12:48, David C. Ullrich <dullr...@xxxxxxxxxxx> wrote:

> On Sun, 8 Mar 2009 08:50:39 −0700 (PDT), Gc <Gcut...@xxxxxxxxxxx>
> wrote:
>
> > On 7 maalis, 06:31, Ben Crowell
> > <crowel...@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx> wrote:
> >
> > > I've been trying to absorb a few ideas from Tarski's work
> > > leading up
> > > to the proof that elementary Euclidean geometry is complete
> > > and
> > > consistent. I'm guessing that the full proof is hopelessly huge
> > > and technical, but I at least want to start by understanding
> > > the interpretation of Tarski's axiomatization.
> > >
> > > He has an axiom schema of continuity. It's basically the
> > > Dedekind
> > > cut construction, but because he wants a first−order theory he
> > > defines
> > > the partitions (A,B) not by quantifications over sets A and B
> > > but by
> > > making an axiom schema in which A and B are defined by
> > > first−order
> > > formulae. The axiom basically says that if every point in A is
> > > to the
> > > left of every point in B, then there exists a point between A
> > > and B.
> > >
> > > Now non−Archimedean models are generally going to
> > > violate this

axiom.

> You've said in the past that your big ambition in life is to prove me wrong about something. You should really be embarassed at the way your attempts to do so all turn out to be ridiculous − if I were you I'd be _very_ careful about proving things I said in that regard.

I have no such ambitions anymore. I

> > There are no non−archimedian models for this theory.

> The proof that there _are_ non−archimedian models is very simple. Your refutations of that proof imply some basic understanding, of what I'm not sure.

OK. Help me with this. How can you say that there are non−archimedian models or even archimedian models, when you can`t define numbers in that theory, eg you can`t extend the language so that it contains numbersand those symbols actually mean something. Now consider models of this theory. There are two relations R and Y, and then there is universe A of a model, models are of the form omega = (A, R ,Y). Now these models have nothing to do with numbers per se. If you wan`t to understand even the original euclidean geometry you must understand that you must in principle be able to construct everything without any numbers in blank paper.

> > There is up to
> > isomorphism exactly one model for each infinite cardinal.

> Supposing that were true, how would that show there are no non−archimedian models?

It is not true, I am sorry. But Aatu knows this stuff and he gave an example where the non isomorphic models have then same language. It is no use to speak about "unique up to isomorphism" if the language are not compatible.

> > That follows
> > from that the theory is complete.

> What?????????????????????
>
> This is the part you should be embarassed about. I gather
> you're actually studying logic.

No I am not. I haven`t ever taken any logic classes and I haven`t read anything about logic since last summer.

> All I know about logic
> is what I learned in a course thirty years ago, but _I_
> know that "elementarily equivalent" is not the same
> as "isomorphic". Jeez.

Yes you are right. I have complete forgot about that.

> Here's something I'm pretty sure is a counterexample.
> The language is FOL with equality, including exactly
> one unary predicate P and no other predicates or
> function symbols.

I thought we were speaking about the theory in OP. In your reply i didn`t notice you saying that you are thinking about some other theory.

> The theory is "there exist infinitely
> many x such that P(x) and there exist infinitely many
> x such that ~P(x)". I can easily give a set of first−order
> axioms that amounts to that, in case it's not obvious
> how to do so.
>
> It seems clear to me that this theory is complete,
> although I haven't given a formal proof. By all
> means explain why it's not.

What this has to do with the original theory?

> The theory certain
> has non−isomorphic models of the same cardinality.
> If c is any uncountable cardinal there are at least
> two non−isomoprhic models of cardinality c:
> One where only countably many elements
> satisfy (the interpretation of) P and one

where only countably many elements
satisfy ~P.

> E.g., A could be the set of infinitesimals and B the positive reals. However, I'm not clear on whether this still applies when A
> and B have to be defined by first−order logical formulae. The first−
> order language in which the formulae have to be constructed can't,
> for example, define the set of all infinitesimals I used in the counterexample above.

> So I can tell that this axiom definitely rules out the rationals as a model, but does it actually rule out the hyperreals or the surreal numbers?

David C. Ullrich

"Understanding Godel isn't about following his formal proof.
That would make a mockery of everything Godel was up to."
(John Jones, "My talk about Godel to the post−grads."
in sci.logic.)

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