Re: Cantor's Argument Skolemized?

Source: http://sci.tech−archive.net/Archive/sci.logic/2009−03/msg00267.html

---

- *From*: Fjodor <frode.bjordal@xxxxxxxxxxxx>
- *Date*: Sun, 15 Mar 2009 13:04:29 −0700 (PDT)

On 15 Mar, 20:25, ross.finlay...@xxxxxxxxx wrote:

> On Mar 15, 10:18 am, Fjodor <frode.bjor...@xxxxxxxxxxxx> wrote:
>
> > > On 15 Mar, 18:04, ross.finlay...@xxxxxxxxx wrote:
> > >
> > > > On Mar 15, 8:40 am, Newberry <newberr...@xxxxxxxxx> wrote:
> > > >
> > > > > On Mar 12, 7:12 am, Fjodor <frode.bjor...@xxxxxxxxxxxx> wrote:
> > > > >
> > > > > > Cantor's Argument Skolemized?
> > > > > >
> > > > > > As all on this list will agree, Cantor's argument is entirely valid.
> > > > > > All, however, will also know about the quandaries this has created in light of the Löwenheim−Skolem Theorem, as stressed by Skolem and known under the heading "Skolem's

"Paradox"".

But it is also a fact that in a ZF−setting one needs a fair amount of separation to carry through Cantor's argument. Separation restricted to first−order conditions A(x) with only x free, and no parameters, will not suffice. For then we may assume that there is a function f from N onto P(N) such that there is no first order condition F(x) such that (x)(xef<=>F(x)), and Cantor's argument is blocked. (It is interesting to see that Skolem in his 1957/58 lectures at Notre Dame only presupposed a non−parametrized version of separation (available as chapter 3 here:http://projecteuclid.org/DPubS?service=UI&version=1.0&verb=Display&). Was he simplifying? When and how did parametrized versions of separation enter the picture?) But with only separation without parameters set theory will be seriously hamstrung, and too weak to serve as a foundation for mathematics.

In the following I offer a couple of ZF−like set theories wherein we explicitly make use of a domain D in stating the axioms. I call everything under consideration, in and out of D, "sets". Some places I make use of the eliminable set−abstract notation. It seems to turn out that we may here block Cantor's argument in a Skolem−like manner in the object language itself. The questions I raise are: (1) Am I making some elementary mistake somewhere? (2) If not (1), (a) is this treated somewhere, or perhaps folklore; (b) is this, in your opinion, of any (i) mathematical, (ii) philosophical or other interest?

I think it would be of tremendous philosophical and mathematical interest if we could construct a set theory without the transfinite stuff, i.e. a set theory that could do all the practical mathematics but either could not prove |P(N)| > |N| or could prove |P(N)| = |N|.

Axiomatic Sketch I:

A0 Ex(x=D) (Existence of the set D)
A(1) (x)(y)(x,yeD=>((z)(zex<=>zey)=>x=y)) (extensionality relative to D)
A(2) (x)(y)(xeD & yex => yeD) (transitivity of D)
A(3) Ex(x=Ø & xeD) (existence of empty set in D)
A(4) Ex(x=W & xeD) (omega exists in D)

A(5) (x)(xeD=>Ey(y=Ux & yeD)) (D is closed under union)
A(6) (x)(y)(xeD & yeD => {x,y}eD) (D is closed under pair)
A(8) All universal closures restricted to D of (z)(zeD=>(Ey)(yeD & (x)(xey<=>xez & A(x))), where A(x) is a first order condition on x that
does not have y free. (D is closed under separation with parameters
from D)
A(9) Replacement is treated analogously with separation.
A(9) (x)(xeD => {y: yeD & y SUBSET OF x}eD) (D is closed under D−restricted Power)

Axiomatic Sketch II:

As Axiom Sketch I but with A(9) replaced by

A(9)' (x)(xeD => {y:y SUBSET OF x}eD)

Let us now rehearse Cantor's argument in the light of Axiomatic Sketch I. Suppose f is a function with Dom(f)=W and Ran(f)=P(W) and such that f is onto P(W). We are then asked to consider the set S={x:xeW & notxef(x)}. If we assume that feD, a contradiction follows. But then

Cantor's argument here only seems to license the conclusion that there is no function *in D* which is a bijection from W to P(W) (or from W onto P(W)). If f is not in D, we do not have enough separation to assert the existence of S. In light of this, it seems that Axiomatic sketch I is consistent with an extension stating that there is a function from W onto P(W), for we need not assume that all sets are in D. (By analogous reasoning, we may add assumptions to the effect that there for all sets X in D is a function f from W onto X.)

Some would here be inclined to complain that A(9) does not capture the "real" power set operation, whatever that might be. Many would e.g. find it objectionable that the postulated f from W onto P(W) is not a member of P(WxP(W)). I have therefore offered Axiomatic Sketch II in order to fend off certain concerns along such lines. Let us say that a set g is a *dijection* of the sets A and B iff

((x)(xeA=>Ey(yeB & <x,y>eg)) & (x)(xeB => Ey(yeA & <y,x>eg)) & (x)(y)(z)(xeA & yeB & zeB & <x,y>eg & <x,z>eg =>

y=z) & (x)(y)(z)(xeB & yeA & zeA & <y,x>eg & <z,x>eg => y=z))

The idea is that a dijection of A and B may have all kinds of other stuff in it besides pairs from AxB. All bijections are dijections, but there are dijections which are not bijections. But it seems fair to say that if there is a dijection from A to B, then A and B have the same cardinality. In the light of Axiom Sketch II, we may now again rehearse Cantor's argument. Suppose there is a dijection f from W to P (W). Now, if we assume that feD we again fall prey to Cantor's conclusion. But f may not be in D. Further, we do not even have enough separation to assert the existence of a bijection f' from W to P(W) such that Dom(f')=W and Ran(f')=P(W). So we may, it seems, postulate the existence of a dijection from W to P(W) without worrying that it should somehow "naturally" be in D.

If this has some merit, it will, it seems, include the one that we may capture Skolem's intuition, that the notion of "uncountability" is only relative, in the object language itself. One may, arguably, maintain the

> view that all sets are in fact
> countable.

In ordinals, there is talk about that in ZFC with the cumulative limit
hierarchy (ZFC's cumulative hierarchy of ordinals through recursivity
into impredicativity, to all sets). Basically for any finite set you
need to have enough values, there are always bigger finite sets. It's
a theorem of ZF − Infinity that for any finite set there is a finite
set containing its elements and other finite sets. That is not to say
for all finite sets is there a finite set containing each of them,
i.e., there is not free induction, but, induction out towards infinity
is from small, finite systems. Because for any, and for each, finite
set there is a bigger finite set, it is possible to use largish finite
sets to represent the sample space, for example ranging in value from
0 to 2 to the thirty−second power
2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2x2.

Then, there is the consideration of how to apply for each of those
discrete values some real value. For that generally integral calculus
can be done without infinities with the consideration of some special
results that help define the number system, the real number system
with the concomitant complex and hypercomplex numbers, naturally.
Then, it helps to maintain the non−linear sampling and evolving
statistics with small terms for local evolution of data. As real
number valued nodes, 0 <−> 1, then there are all kinds of functions,
including special ones, with that domain in terms of the rest of the
continuum of real numbers.

So, how can there be countable reals, with the usual
properties of
real numbers, when reals are an infinite set in ZF that are
uncountable? As a finite sample of what is an infinite space,
finite
constructions of the reals see some special functions in the
digital.
Generally they are quantized.

Then, with these modern fundamental expansions in analysis
towards new
tools with symbolic reductions in reversible systems,
mathematics is
generally applicable. Then, ZF−Infinity has all finite sets,
but
constructions of the real numbers are countable which an
adherent of
ZF, with its intuitive axiom about infinity that there is a set
of at
least infinitely many items, in terms of other sets, would
have
incomplete models of the reals, yet that is a known issue with
ZFC
also for different reasons. Various axiomatizations of
continuum
separately simplify computation.

Regards,

Ross Finlayson Skjul sitert tekst

Vis sitert tekst

If I understand you correctly, your point relates to what can be done in ZF minus infinity. I do not quite see the relevance of this to my posting. Maybe there is something I fail to understand.

Add your own axioms and see if they are consistent.

That's the (a) point of reversible and also forward logic, Goedel has completion theorems too. (Also Goedel writes out incompleteness theorems.)

So, add some other definition, hopefully in the smallest possible amount of logical primitives, to define infinity, like the largest number. Then it's reverse in separability. Or, just say it doesn't exist.

Regards,

Ross Finlayson Skjul sitert tekst

Vis sitert tekst

I would need to see what you have in mind in detail to make a judgement as to its viability (and/or relevance)..
.