Re: Fast solution to very small eigenvalue problem

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From: Peter Spellucci (*nospamspellucci_at_fb04373.mathematik.tu−darmstadt.de*)
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In article <PCn*9dSnq@news.chiark.greenend.org.uk>,
Mark Mackey <markm@chiark.greenend.org.uk> writes:
>Hi all.
>
>I need to find the eigenvector corresponding to the largest eigenvalue
>of a 4x4 matrix very quickly (because I'm doing it on hundreds of
>thousands of 4x4 matrices). The current code I'm maintaining has a
>simple Jacobi solver, which is (a) slow (it only does 30K matrices/s on
>my PC), and (b) probably overkill, as it returns all of the
>eigenvectors. I've vaguely looked at LAPACK etc, but those routines are
>AFAIK optimised for good performance on large matrices, not small ones.
>
>Does anyone have any suggestions as to the most efficient way to solve
>this problem? Extreme accuracy is not required. 4x4 is probably small
>enough that there's an analytic solution :).
>
>−−
>Mark Mackey
>"The determined Real Programmer can write Fortran programs in any language."
> − "Real Programmers don't use Pascal"

you did not mention it by from "jacobi" I conclude −> symmetric.
hence:
1) transform to tridiagonal form. don't use LAPACK, write this yourself, just
3 givens rotations.
2) use bisection on −norm(A),norm(A) for example the infinity norm =
max row sum of abs−values.
bisection means counting the negative pivots in the lu−decomposition of
the tridiagonal matrix −mu*I, without pivoting, replacing a zero pivot by eps>0
this disturbs the eigenvalues by at most eps. if the number of negative pivots is
<= 3 at mu then mu = new lower bound, otherwise mu = new upper bound,
until sufficient precision is attained.
3) solve now (A−mu I)*x =0 using complete pivoting and setting artificially x(4)=1,
substituting back then.
done
hth
peter