

## Re: better than a quartic?

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On 3 Jul 2004 07:17:07 -0700, Carlo\_02@libero.it (Carlo) wrote:

> *I have to solve this set of equations:*  
>  
>  $t5 = t1 + (t1-t2)^2 / ((t1-t3)*(Pa-1))$   
>  
>  $t1 = t5 + (t5-t4)^1 / ((t5-t3)*(Pb-1))$   
>  
> *t1 and t5 are unknown. t2, t3, t4, Pa, Pb are known.*  
> *In practice, these properties are also true:*  
>  $0 < t2 < t3 < t4$   
>  $0 < Pa < 1$   
>  $0 < Pb < 1$   
>  
> *With some manipulation, I can turn the equations above into a quartic,*  
> *and solve it numerically (I know there is also a symbolic solutions to*  
> *a quartic, however, the coefficients of this quartic are already a*  
> *mess, never mind the symbolic quartic solution...). Once I get t1 or*  
> *t5 this way, I can find the other easily by applying one of the above.*  
>  
> *Unfortunately, with the numbers I have to play with, the quartic is*  
> *relatively a mess.*

Probably the best that can be done algebraically is try and solve for the unknowns in terms of a parameter that satisfies a simpler quartic than maybe the one you found.

For typing convenience I've relabelled your variables and unknowns as follows:

$x, y, a, b, c, d, e := t5, t1, t2, t4, Pa - 1, Pb - 1, t3$

so your equations become:

$$x - y = (x - b)^2 / (d.(x - e)) = (y - a)^2 / (c.(y - e))$$

taking:

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$$u, v, A, B := (x - b)/d, (y - a)/c, (b - e)/d, (a - e)/d$$

these become:

$$d.u - c.v + b - a = u^2 / (u + A) = v^2 / (v + B)$$

The latter pair are equivalent to:

$$B((2A/u + 1)^2 - 1) = A((2B/v + 1)^2 - 1)$$

So taking:

$$2A/u, 2B/v = s, s.t$$

gives either  $s = 0$  or a linear equation in  $s$  which implies the following general parametrization of  $u, v$  in terms of  $t$ :

$$u = t.v = -A.(A.t^2 - B)/(A.t - B)$$

and plugging that into " $d.u - c.v \dots =$ " gives the quartic:

$$(A.t^2 - B)^2 + (A.c.t - B.d + b - a).(t - 1).(A.t^2 - B) - B.(b - a).(t - 1)^2 = 0$$

This is a homogenous quadratic in  $A.t^2 - B$  and  $t - 1$ . But unfortunately if  $A.c \neq 0$  there is a pesky  $t$  in the middle coefficient, and this means it can't be solved directly as a pair of quadratic steps.

(If you decide to use the preceding, it would be a good idea to check the algebra!

Cheers

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John R Ramsden (jr@adslate.com)

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"Eternity is a long time, especially towards the end."  
Woody Allen