

Re: quadratic minimization: analytical solutions

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In article <1104478990.210548.7520@z14g2000cwz.googlegroups.com>,

y_granik@yahoo.com writes:

> *Consider quadratic non-convex optimization*

> *problems with simple bounds:*

>

> $f(x) = xQx + bx \rightarrow \min$

> $0 \leq x \leq 1$

>

> *When x has only one component, we get*

>

> $f(x) = a*x*x + b*x \rightarrow \min$

> $0 \leq x \leq 1$

>

> *This is pretty simple problem*

> *and can be solved analytically.*

> *If $a > 0$, then problem is convex and we just have to*

> *compare 2 or 3 values*

>

> $f(0)$, $f(1)$, and $f(-b/2a)$, if $0 \leq -b/2a \leq 1$

>

> *and identify the smallest one.*

> *If $a < 0$, then the problem is concave*

> *and we have to compare*

>

> $f(0)$ and $f(1)$

>

> *to find min.*

>

> *The question: does this problem*

> *have analytical solutions in dimensions*

> *high than 1? In particular,*

> *for 2 dimensions, when $x = (x_1, x_2)$,*

> *can we solve this analytically?*

>

>

> *This feels like a fundamental*

> *problem of quadratic programming,*

>but I cannot find it in the books I
>have.
>
>Please help.
>
>Happy New Year!
>
>Yuri Granik
>

no. nonconvex quadratic optimization is np hard.
even in case $n=2$ you could have any situation concerning the minimizer.
make a little sketch e.g. with $Q = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and any b , with the maximum now
at $-b/2$. depending on the position of $-b/2$ relative to the unit box
you could have several local (nonglobal) minimizers, with $b = [-1, -1]$ four
global minimizers exactly at the four vertices etc etc
hth
peter