

Re: Regularly populating the line – revised version

Source: <http://sci.tech-archive.net/Archive/sci.math.num-analysis/2005-01/0487.html>

From: Gordon Sande (g.sande_at_worldnet.att.net)

Date: 01/30/05

Date: Sun, 30 Jan 2005 15:14:55 GMT

Lord Snooty wrote:

```
> Due to a typo, one more time with feeling, and less C-stylistic too:
>
> for (i=a=0; i < n; i++) {
> a = a + p;
> if (a >= n) {
> // fill at position i
> a = a - n;
> }
> // else leave position i unfilled
> }
>
> It's trivial to analyse this for  $(n \bmod p) = 0$  of course.
> I'm only interested in nonzero  $(n \bmod p)$ .
>
```

The first observation is that a is just $i \cdot p \bmod n$. Then one wants to know what make the if condition true. It is true whenever the value of $i \cdot p / n$ (integer divide) changes. Otherwise stated as $(i-1) \cdot p / n \neq i \cdot p / n$. This starts to look like the algorithm for drawing a diagonal line on an integer lattice, which is Bresenham's algorithm if you are doing computer graphics. A full citation is left as an exercise for the reader. Some of the dates will be in the prehistoric period BG (Before Google).

The curious handle does not suggest a strong academic interest so one might even guess that the original problem is computer graphics. The ancients did have considerable wisdom and the even more curious habit of recording much if it in books. So the obvious thing is for Lord Snooty to try reading some books.

```
>
> "Lord Snooty" <bonzo@dog.com> wrote in message news:...
>
>>It's not too uncommon to want to populate  $n$  bins with a given number  $p$  of
>>objects, such that they are as regularly spaced as possible – we seek to
```

```
>>avoid
>>bunching, in other words. For example, if  $n=8$  and  $p=2$ , then trivially we'd
>>seek one of the cyclic permutations of  $PxxxPxxx$ , where  $P$  represents one of
>>the
>> $p$  objects and  $x$  represents an unfilled bin.
>>
>>I came across the following very efficient algorithm recently, which works
>>on
>>all tested integer pairs  $(n,p)$ ,  $n \geq p$ . Trouble is, I have difficulty
>>proving
>>that it will always emit exactly  $p$  objects. I'd appreciate any attempts at
>>proving this. I'd also like to know if it has a name, because I've never
>>come
>>across it before. I can't find it in Knuth, for example, because I can't
>>find
>>a good name for it with which to do an index search. Here's the algorithm
>>(C-code style):
>>
>>for (i=a=0; i < n; i++) {
>> a += p;
>> if (a >= n) {
>> //fill at position i
>> a -= nN;
>> }
>> // else leave bin unfilled
>>}
>>
>>
>
>
>
```