Legendre,Chebychev polynomials

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In the following, let us denote :
SUM{A_k}:=A_1+A_2+...+A_n ; (n >= 1)
P_n(x)=k_n*[(x^2−1)^n]^{(n)} , P_n(1)=1 , be the Legendre polynomial
having the roots a_1,a_2,...,a_n ;
U_n(x)=sin((n+1)*arccos(x))/((n+1)*sqrt(1−x^2)) , U_n(1)=1 , be the Chebychev
polynomial of second kind with the roots
b_k=cos(k*pi/(n+1)) , k in {1,2,...,n} .

[A.] I need elementary proofs of following identities /if true/:

(1) P_n(b_k)=SUM{(b_k)^n *U_n((b_k)^2)}

(2) U_n(a_k)=−SUM{(b_k)^n *U_n(a_k*b_k)}

It's true that
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(1−2) P_n(x)−U_n(x)=SUM{(b_k)^n*U_n(x*b_k)} ??
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[B.] Let c_k=cos((2k−1)pi/(2n)) , k in {1,2,...,n}, and denote

Q_{2n}(x):= SUM{ P_n(x^2+(1−x^2)*c_k) } .

It's true that Q_{2n}(x) has only double roots ??