Re: Help with eigenvalue decomposition

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- *Date*: Thu, 6 Oct 2005 04:29:41 +0200

Let A be full−ranked, symmetric, K^(n x n).
Let b_1 be eigenvector of A, then c_1*b_1 (c element of K) is also
eigenvector of A.

=> If A can be represented as A = B*D*inv(B), then A can be also represented
as A = B~*D*inv(B~), where B~ = [c_1*b_1 | c_2*b_2 | ... | c_n*b_n]. As you
can see, you can "use" for every eigenvector an own "c".

=> If A is diagonalizable, the diagonalization is not unique.

But in your case, you have some stricter assumptions.

> B is a unitary matrix (inv(B)=B^T) and D is the diagonal matrix that
contains the eigen values of A in increasing order. The elements of B are
orthonormal eigenvectors of A.

What I showed you beyond, is now no longer possible.
Let P be unitary. P~ = c*P is no longer unitary, c <> 1, so you cannot use
inv(P~)=(P~)^T.

We could think, now the diagonalization is unique: If we have the
eigenvalues in 1 matrix D, we have 1 Matrix B.

But you think, there could be a second solution with the same matrix D, but
another unitary matrix, E, that should be a permutation of B.

> Is it correct to say that F=D and E=Perm(B) i.e. the columns of E are
permutation of columns of B?

Ok, let me do a "short" 2 x 2 counter−example.

B=[b11, b12; b21, b22]
D=[d1,0; 0, d2]
B^T=[b11, b21; b12, b22]

B*D =
[b11*d1, b12*d2;
b21*d1, b22*d2]

B*D*B^T=
[b11^2 *d1 + b12^2 *d2, b11*b21*d1 + b12*b22*d2;
b21*b11*d1 + b22*b12*d2, b21^2 *d1 + b22^2 * d2]

If you permutate B, you get:

E=[b12, b11; b22, b21]
F=D=[d1,0; 0, d2]
E^T=[b11, b21; b12, b22]

E*F =
[b12*d1, b11*d2;
b22*d1, b21*d2]

E*F*E^T=
[b11^2 *d2 + b12^2 *d1, b11*b21*d2 + b12*b22*d1;
b21*b11*d2 + b22*b12*d1, b21^2 *d2 + b22^2 * d1]

As you can see, it is almost the same. The only difference is, that d1 and d2 changed their places.
But this is a contradiction to your assumption F=D. It would be only true, if F is "the right permutation" of rows of D.

Best wishes,
Sumit

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- ***Follow−Ups***:
  - ***Re: Help with eigenvalue decomposition***
    - *From:* Sumit Mitra

- ***References***:
  - ***Help with eigenvalue decomposition***
    - *From:* Amit Singh

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