Re: Optimization for a positive definite constraint

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Hiu Chung Law wrote:

> Let X be a n by n matrix.
>
> The goal is to minimize a real−value function f(X)
> subject to the constraint that X is symmetric, and X is positive definite.
> (The function f is not defined for X that is not positive definite).

If f(X) is linear in the elements of X, then you have a semidefinite programming problem. The SDP formulation can also include linear equality and inequality constraints on the elements of X. Also, many problems that don't appear at first to be SDP's can be reformulated as SDP's. For example, many eigenvalue optimization problems can be rewritten as SDP's. Quite a bit of work has been done on solving SDP's, and there are several open source and non−free software packages for solving such problems. You should certainly make use of the work on SDP's if possible.

If f(X) is a more general non−linear function, then you have a nonlinear semidefinite programming problem. Much less research has been done on such problems, but there are certainly some ways to approach these problems.

For example, if f(X) is convex, then one approach is to construct an outer approximation to f(X) using linear constraints derived from the gradient of f(X). You start with a solution X0, find the gradient of f at X0, and use it to construct a linear function f0(x) that underestimates f(X). Solve a linear SDP with this constraint and obtain a solution X1. Repeat.

One important issue concerns the nature of the function f(X). Is your function convex or nonconvex? Is it differentiable? Does it have some particular structure, such as being a function of the eigenvalues of X? Can X be easily expressed in terms of a smaller number of parameters? The answers to these questions could help in selecting an appropriate approach to solving the problem.

Another important question has to do with the size of the problem. How big is n?

> I am now treating this as an optimization problem with n^2 variables, and the gradient and the Hessian can be computed.
>
> The symmetric constraint can be handled easily because it is linear. However, the constraint that X should be positive−definite is more difficult to handle. Right now, I simply force my line−search procedure to reject any X that is not positive definite. However, this does not seem to be robust.

This is not a good way to go for any constrained optimization problem.

> I have also re−parameterized X by X = F * F', but the Hessian with respect to F is much more nasty than the Hessian with respect to X.

This is a well know approach, but it doesn't seem like a good choice in your case.

> I thought something related to how to handle bound constraint may help, but I cannot find anything similar. Also, this is quite different from semi−definite programming.... Do you have any advice on how to tackle this type of problem?

I would suggest that you start by reading up on "nonlinear semidefinite programming." A search on google scholar gives 53 hits, including several that look quite likely to be useful. For example, there's

A Global Algorithm for Nonlinear Semidefinite Programming
SIAM Journal on Optimization
Volume 15 , Issue 1 2005
Pages: 303 − 318
Authors: Rafael Correa and Hector C. Ramirez

You might also tell us more about your particular problem so that we could suggest methods appropriate to your f(X).

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