

Runge Kunta on SDE

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- *From:* David Langevin <langevin_david@xxxxxxxxxxxx>
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Dear readers,

I want to apply the Runge Kunta fourth order scheme on the following SDE:

$$dX = aXdt + bXdW$$

where a,b are real, $dW = \sqrt{dt}N(0,1)$ is a wiener process.

When b=0 in my program all is fine, the solution is the same as in the deterministic case (Hopefully !), the problem I have is in the stochastic part, the scheme doesn't converge to the true solution:

$$X = X_0 \exp\left(\left(a - \frac{b^2}{2}\right)t + bW\right)$$

where X_0 is real.

Here is the small program that compute RK4:

```
function RungeKuntaApprox = RK4(N, M, X0, a, b, dt, dW)
```

```
L = (floor(1/dt)); % time step
RungeKuntaApprox = zeros(N*M,(floor(1/dt)));
Xtemp = X0;

for j = 1:L
    W = sum(dW(:,1:j),2);
    k1 = a*Xtemp*dt + b*Xtemp.*W;
    k2 = a*(Xtemp + 0.5*k1)*dt + b*(Xtemp + 0.5*k1).*W;
    k3 = a*(Xtemp + 0.5*k2)*dt + b*(Xtemp + 0.5*k2).*W;
    k4 = a*(Xtemp + k3)*dt + b*(Xtemp + k3).*W;
    Xtemp = Xtemp + 1/6*(k1 + 2*k2 + 2*k3 + k4);
    RungeKuntaApprox(:,j) = Xtemp;
end
```

Thank's for your contribution !

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