Re: Recommendation for numerical differentiation formulas?

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- *From*: spellucci@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx (Peter Spellucci)
- *Date*: Mon, 17 Jul 2006 13:37:54 +0000 (UTC)

In article <1153137085.886668.152400@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
1940LaSalle@xxxxxxxxx writes:

> Peter Spellucci wrote:
>
> > In article <1152885890.796128.65800@xxxxxxxxxxxxxxxxxxxxxxxxxxxx>,
> > 1940LaSalle@xxxxxxxxx writes:
> > >
> > >Peter Spellucci wrote:
> >
> > no, flip in a geometric sense right to left. means
> >
> > f'(x(n+1))=(1/(12*h))
> > *(−25*f(x(n+1)+48*f(x(n))−36*f(x(n−1))+16*f(x(n−2))−3*f(x(n−3)))
> >
> > you need these special one sided formulae only for the first data point
> > (I named it x(0)) and the last one x(n+1) because these are not covered by
> > the 3 point compact scheme. all other derivatives come out from the
> > tridiagonal solve, and you are right: the right hand side is made up from the
> > table and looking at the indices you see: f(x(0)) up to f(x(n+1)) are needed
> > there and give you u(1) up to u(n) which correspond to f'(x(1)) up to f'(x(n))
> > good luck
> > peter
>
> In thinking about this scheme, it seems I have a few more questions
> about the mechanics of it. For the sake of discussion, let's say the
> data involved have a total of j points. Then the right side of the
> equation would appear to be a matrix (that is, a vector) with one
> column and j−1 rows (there are no rows corresponding to x(0) and x(j)
>). The left side is the tridiagonal matrix we've mentioned before. I
> may be missing something fundamental here, so please bear with me: when
> one inverts the matrix on the left and does the appropriate matrix

> multiplication, one gets a solution vector (again, one column with j−1
> rows)−−at least, I believe that's the case. In that solution vector,
> the first row would yield the value of u(x(1)), the second, u(x(2)),
> etc., up to u(x(j−1)), correct? (What I know of matrix

yes

> algebra/manipulation has been acquired purely as needed; I've never
> taken a formal course.)

this is a linear system of equations with a tridiagoanl matrix
and you never invert the matrix, you solve the system by standard
Gaussian elimination with amounts in just some j arithmetic operations
and hence is extremely cheap.

> Also: I'd like to verify that all of the coefficients on the left side
> have the same sign. Unless I set up the matrices incorrectly−−not out
> of the question−−I would expect that the results would all have the
> same sign if the tabulated function is monotonic with no local maxima
> or minima within the range of the independent variable.

yes, if your function data are not subject to noise. noise will
blow up by a factor 1/h, hence this may destroy these properties
if it is too large

> Last but not least: what's the reference for the one−sided formulas?
> I'd like to see/read more.
>
> Thanks again for your patience: this is very valuable.

the linear system is from

Quarteroni, Alfio; Sacco, Riccardo; Saleri, Fausto
Numerical mathematics. (English)
Texts in Applied Mathematics. 37. New York, NY: Springer.

the one sided formula is from
Schmeisser, Gerhard; Schirmeier, Horst

Praktische Mathematik. (German)
de Gruyter Lehrbuch. Berlin−New York: Walter de Gruyter

but there should be other sources containing it it is obtained
simply by Taylor's formula for f(x+k*h) k=0,1,2,3
resp. f(x−k*h) and combining these values linearly such that the f'(x)
obtains with error term h^4

hth
peter

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