Re: eigenvector for a 3−by−3 symmetric real matrix

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- *Date*: 5 Apr 2007 03:17:33 −0700

On Apr 5, 11:05 am, "Jeremy Watts" <jwatts1...@xxxxxxxxxxx> wrote:

> "agou" <agou....@xxxxxxxxx> wrote in message
> news:1175746181.292952.303120@xxxxxxxxxxxxxxxxxxxxxxxxxxxxx
>
>> hi, there
>> i wanna calculate the eigenvector to the smallest eigenvalue for a 3−by−3 symmetric positive definite real matrix. i know it might sound silly. however, i wanna put this stuff running on hardware. therefore, analytic result would be best. however, it's not trivial. therefore, maybe an iterative but fast converging algorithm will be preferred. QR looks like too complicate either. does anyone have any better idea or book i could refer to?
>> thanks a lot! regards zhaoyi

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Assume that
A=
[a_{1,1} a_{1,2} a_{1,3}]
[a_{2,1} a_{2,2} a_{2,3}]
[a_{3,1} a_{3,2} a_{3,3}]

is an arbitrary 3 x 3 real matrix.
Then the eigenvalues r_1,r_2,r_3 are the roots
of charcteristic equation
det(A−r*I)=0 which is equivalent to

P(r):=r^3−s*r^2+u*r −d =0

where
s=tr(A):= a_{1,1}+a_{2,2}+a_{3,3}

u: =sum of three 2x2 determinants=
|a_{2,2} a_{2,3}|
|a_{2,2} a_{2,3}|+

|a_{1,1} a_{1,3}|
|a_{3,1} a_{3,3}|+

|a_{1,1} a_{1,2}|
|a_{2,1} a_{2,1}|

and d:=det(A).

Further assume that r_1,r_2,r_3 are real,
e.g. in case when A is symmetric. More
precisely consider

r_3 =<r_2 =<r_1 .

Find D:=2*s^2−6*u . Note that D>= 0 .

Denote x:= (r_1+r_2+r_3)/3 = s/3 .

Then

sqrt(D/2) =< |r_3−r_1|=< sqrt(2*D/3)

Q(1):=x + sqrt(D*0.5)/3 =< r_1 =< S(1):=x+sqrt(2*D)/3

Q(2):=x−sqrt(D*0.5)/3 =< r_2 =< Q(1)

Q(3):=x−sqrt(2*D)/3 =< r_3 =<Q(2)

In order to find the eigenvalues apply an iterative method,
like Newwton, that is

x_{n+1}=x_n − f(x_n)/f'(x_n) , n=0,1,..., x_0:=r_0 .

Select as starting points r_0 one of numbers

{S(1),Q(1),Q(2),Q(3) }.
Let us note that, according to Fourier,

r_0 is "good selected " when f(r_0)*f''(r_0) > 0 .

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