Re: Schmidt decomposition for operators?

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- *From*: Robert Israel <israel@xxxxxxxxxxxxxxxxxxxxxxxxxxxxxx>
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Javier Almeida <javialmeida@xxxxxxxxx> writes:

> Hi everybody,
>
> let O be a matrix with dimension n x n. I would like to write this operator as a sum of kronecker products, i.e,
>
> O= Sum_{i=1}^{i=k} lambda_i*kron(A_i,B_i)
>
> with A_i and B_i operators of size (ma x ma) and (mb x mb), holding that ma*mb=n. In particular I would like to obtain such a decomposition that requires the minimum number of sums, that is, a decomposition such that k is as low as possible. Does anybody know a systematic way to obtain one possible set operators A_i and B_i and the coefficients lambda_i for a given operator O?
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> Thanks,
> Javi.
>
> P.S: the kronecker product of two matrices is the usual tensorial product of two matrices.

I suppose ma and mb are given, so you don't allow the trivial solution
k=1 with ma = 1, mb = n, A_1 = (1) and B_1 = O.

The matrix structure is a distraction: what you really are looking at
is a vector V of N = n^2 components, which you want to write as a sum
of tensor products of vectors with Ma = ma^2 and Mb = mb^2 components:
V = sum_{i=1}^k A_i \tensor B_i.
Let V_j, j=1..Mb, be the Ma−component vectors corresponding to the
individual components of the B vector, so that
V_j = sum_{i=1}^k (B_i)_j A_i.
Thus the V_j are all in the span of A_1, ..., A_k. In particular, k
must be at least the dimension of the linear span of V_1, ..., V_{Mb}.
Moreover, we can attain this k by letting the A_i form a basis of
this linear span.
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