

sci.math.research: expand the right-hand side on the basis of the eigenvectors of the matrix

expand the right-hand side on the basis of the eigenvectors of the matrix

Source: <http://sci.tech-archive.net/Archive/sci.math.research/2004-09/0142.html>

From: Taglit (taglit_is_at_yahoo.com)

Date: 09/19/04

Date: 19 Sep 2004 13:45:01 -0400

Hello!

I have a problem: $Ax=b$, where

$$A = \begin{bmatrix} 1001 & 1000 \\ 1000 & 1001 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$b = \begin{bmatrix} 2001 \\ 2001 \end{bmatrix}$$

a small change of b $db = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Could somebody explain why the small variation in the right hand side produces large variations in the solution?

Here is a hint: expand the right-hand side on the basis of the eigenvectors of the matrix.

SO. I found eigenvectors: $x_01 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $x_02 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

It forms an orthogonal basis, because the matrix A is symmetric.

Then, when we expand the right hand side in the basis of the eigenvectors, then we have:

$$b = c_1 * x_01 + c_2 * x_02, \text{ Hence } c_1 = 2001 \text{ and } c_2 = 0$$

But, when we use $(b + db) = c_1 * x_01 + c_2 * x_02$, then $c_1 = 2001.5$ and $c_2 = 0.5$

So, in the first case $c_2 = 0$ and in the second $c_2 = 0.5$.

sci.math.research: expand the right-hand side on the basis of the eigenvectors of the matrix

I believe that the answer to the question: "why the small variation in the right hand side produces large variations in the solution? should be somewhere here...

But what is a name of a theorem or a method which will help me to answer such question.

Sincerely,
Taglit