Fibonacci connection between Huffman codes and Wythoff array

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Fibonacci connection between non−decreasing sequences of positive integers
producing maximum height Huffman trees and the Wythoff array has been proved.

http://arxiv.org/abs/cs.DM/0410013

Abstract
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A non−decreasing sequence of positive integer weights
P ={p[1], p[2],..., p(n−i)[n]}
is called k−ordered if an intermediate sequence
P(i) ={p(i)[1], p(i)[2],..., p[n]}
of weights produced by Huffman algorithm for initial sequence P on i−th step
satisfy the following conditions:
p(i)[2] = p(i)[3], 0 <= i <= k;
p(i)[2] < p(i)[3], k+1 <= i <= n−3.
Let T be a binary tree of size n and M=M(T) be a set of such sequences of
positive integer weights that the tree T is the Huffman tree of P (|P|=n).
A sequence Pmin of n positive integer weights is called a minimizing sequence
of the binary tree T in the class M, if Pmin produces the minimal Huffman cost
of the tree T over all sequences from M.
Fibonacci related connection between minimizing k−ordered sequences of
the maximum height Huffman tree and the Wythoff array [Sloane, A035513:
http://www.research.att.com/cgi−bin/access.cgi/as/njas/sequences/eisA.cgi?Anum=035513,
http://www.research.att.com/~njas/sequences/classic.html]
has been proved.
Let M[n,k] (0 <= k <= n−3) denote the set of all k−ordered sequences of size n for which the Huffman
tree has maximum height. Let F(i) denote i−th Fibonacci number.
Theorem: A minimizing k−ordered sequence of the maximum height Huffman tree
in the class M[n,k] is Pmin[n,k] = {p[1],..., p[n]} where
p[1] = 1,
p[2] = F(1),
p[3] = F(2),
...,
p[k+2] = F(k+1),
p[k+3] = F(k+2) = w[F(k+2),0],

p[k+4] = w[F(k+2),1],
p[k+5] = w[F(k+2),2],
...,
p[n] = w[F(k+2),n−k−3],

where w[i,j] is (i,j)−th element of the Wythoff array. The cost of Huffman trees for those sequences has been computed. Several examples of minimizing ordered sequences for Huffman codes are shown.

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