Re: new products in geometric algebra

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"Mark Adams" <markjadams@lycos.com> schrieb im Newsbeitrag
news:clar00$1qc$1@dizzy.math.ohio−state.edu...

> *One question I have: has anyone come up with a clean definition in terms*
> *of Clifford (geometric) multiplication for the "inner product" as*
> *defined on the exterior algebra to define the Hodge star? For A and B*
> *k−blades (i.e. the exterior products of k vectors A_i and B_j) this is*
> *the construction*
>
> *<A, B> := det(<A_i, B_j>)*
>
> *with the result defined to be zero for blades of different grade*
> *(composed of different numbers of vectors). Even an operation that*
> *coincided with this for two k−blades would be of interest. I've looked*
> *into various operations including:*
>
> *− "inner product" A.B := <AB>_|j−k| where A and B are j− and k−vectors*
> *− "commutator product" A x B := (AB − BA)/2*
> *− "contraction" A J B := (A /\ (Bw))w^−1 where w is the volume element*
> *(pseudoscalar) and J is supposed to be a backwards "L"*
>
> *but none seem to do the job.*

I think what you are looking for is <A^+,B>_0 ,

(where ^+ is reversion).

You can check that this gives the desired result by looking at a convenient orthonormal basis.

You can explicitly derive this expression by noting that the exterior bundle supports two mutually supercommuting copies of the Clifford algebra that you have in mind, constructed from sums and differences of operators of exterior and interior multiplication, respectively. Noting that the latter are mutual adjoints with respect to the Hodge inner product and using the symbol map you can then derive the above (up to a global sign, depending on some conventions) from the Hodge inner product.

If you want to see the details have a look at pp.279 of
http://www−stud.uni−essen.de/~sb0264/sqm.html .

> A related item: I've been putting the definition of the Hodge star in
> terms of geometric algebra operations
>
> *A = ((w^−1)A)^+ where w^−1 is the inverse of the volume element
> (pseudovector), and ^+ indicates reversion
>
> into some more usable (for what I'm doing) forms. Can anyone verify
> these results? Here A is a k−blade, w is the volume element
> (pseudoscalar), and s is the number of negative signs in the signature
> of the inner product defining the Clifford algebra.
>
> *A = (−1)^(k(k−1)/2 + s) Aw
> *A = (w^+w)(A^+)w

This is also discussed at the above reference.

I have found it very helpful to pass between Clifford calculus and exterior calculus this way when dealing with supersymmetric quantum systems. For instance hep−th/0311064 and math−ph/0407005 makes use of this.