rapidly converging rational sqrt

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Below is a description of an algorithm which, with
each iteration, will double the number of significant
digits in the computation of a rational square root
approximation.

I do not know if this algorithm is new, but I found
it interesting nonetheless. Lisp code for implementing
this algorithm can be found at:

http://thegreves.com/david/sqrt/sqrt.html

If by convention we say that:

s = isqrt(C)

d = C − s^2

Then the square root of C can be expressed as as the infinite
continued fraction:

```
       d
s + −−−−−−−−−−−−−−−−−
            d
    2s + −−−−−−−−−−−−
                d
        2s + −−−−−−−

            2s + ..
```

We designate the tail of this continued fraction using

e(n) = nth error term

and we say that the nth error term of the continued fraction
representation has the form:

```
           d
e(n) = −−−−−−−−−−
```

$$2s + e(n+1)$$

Although we sometimes drop the subscript on the error term for notational convenience.

A pretty good first order approximation for a square root can be computed as follows (even allowing $e(1)$ to be zero):

$$\mathrm{sqrt}(C) \approx s + \frac{d}{2s + e(1)}$$

Without proof, we claim that a generalized expression for a partial evaluation of our continued fraction can be represented as:

$$\mathrm{sqrt}(C) = s + \frac{A + Be}{C + De}$$

It is easy to see that the first order approximation given above is an instance of this expression when $A = d$, $B = 0$, $C = 2s$, $D = 1$. The generali