monoidal enriched natural transformations

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From: Agustí Roig (*soquiso_at_hotmail.com*)
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Hi.

I have a problem which, it seems to me, requires the notion of enriched natural transformation between enriched monoidal functors, but I haven't been able to find a good reference for it.

I've taken a look at the books of Borceux (Handbook of categorical algebra), Kelly (Basic notions of enriched category) and the appendix in Levine's "Mixed motives", but they all end just where I need them, or even long before that point.

More precisely, the situation I've encountered seems to be the following. I have:

− Two monoidal (symmetric) categories:

V and W .

− A couple of monoidal (symmetric) functors between them:

S, T : V −−−> W

− A monoidal natural transformation between S and T :

\omega : S ===> T

And here is where my problems begin.

There is a well−known notion of what is a V−functor between V−categories and what a V−natural transformation is.

My first need is to understand what an S−functor between a V−category C and a W−category D should be:

F : C −−−> D

I didn't find this thing in the literature, but I expect it ought to be something like a V−functor, but with a family of morphisms in W

\lambda_{XY} : S[X,Y] −−−> [FX,FY]

for every pair of objects X, Y in C . (Here the square brackets [,] stand for the objects in V and W of "morphisms" of C and D , and I'm leaving aside units and commutative isomorphisms for the moment.)

This seems reasonable to me, since (a) is the situation I have in the "real" world and (b) if I put S = id_V , I find the definition of a V−functor.

Next, I would need the notion of an "\omega − natural transformation" and I think this should be something like a V−natural transformation between an S−functor F : C −−−> D and a T−functor G: C −−−> D, but placing at the beginning of the commutative diagram which defines a V−natural transformation an arrow like

\omega_{[X,Y]} : S[X,Y] −−−> T[X,Y] .

Assuming that this is ok, I should also need to understand what might be the definition of a "monoidal \omega − natural transformation".

That is to say, C is a monoidal V−category, D is a monoidal W−category, F is a monoidal S−functor and G a monoidal T−functor: what is a monoidal natural transformation between F and G , over the monoidal natural transformation \omega : S ===> T ?

I've drawn a couple of commutative diagrams that should appear in the definition of such a construct, but I feel I could be forgetting a dozen more. Any references for it?

Unfortunately for me, Kelly's book ends before this point: it explicitely says: "is our decision not to discuss the 'change of base−category' given by a symmetric monoidal functor V −−−> W". Has someone else done the job after Kelly's book?

Agustí Roig