

sci.math.research: Re: Partitioning 4 space with ultraskew lines, and the three body problem.

Re: Partitioning 4 space with ultraskew lines, and the three body problem.

Source: <http://sci.tech-archive.net/Archive/sci.math.research/2004-11/0022.html>

From: Robert E. Beaudoin (*reb_at_sens.com*)

Date: 11/03/04

Date: Tue, 02 Nov 2004 22:31:03 -0500

Richard L. Peterson wrote:

- > *Suppose we are in 4 space. Two lines are "skew" if they do not lie in*
- > *the same plane(2 space). Skewness is a binary relation on lines. Now*
- > *two skew lines will lie in the same 3 space. But it is possible for*
- > *three lines to not lie in the same 3 space. This is a ternary relation*
- > *on lines. I don't know if there's a name for this relation, so let's*
- > *call it "ultraskewness" until we find out.*
- >
- > *1) Can one foliate, or at least partition, 4 space with lines that*
- > *are tripletwise ultraskew? I mean EACH 3 line subset of the partition*
- > *must not lie in the same 3 space.*
- >
- > *2) In the three-body problem, one could approximate the trajectory*
- > *of each of the 3 masses as a straight lines until they became close*
- > *enough to each other for their gravitation to have an appreciable*
- > *effect. I think much of the work on this problem assumes all three*
- > *trajectories lie in the same plane.(The restricted 3-body problem.)*
- > *But some work has been where the trajectories are not coplanar.*
- > *Wouldn't it be fun to explore the three-body problem when the*
- > *trajectories are not cospatial?*
- >
- > *Richard Peterson, CSU Sacramento*
- >

I don't know about the three-body problem, or about foliations, but I think one can partition 4-space into "ultraskew" lines without much trouble. The proof is via a transfinite induction of c (the cardinality of the continuum) many steps; at step k one considers the k -th point p in a fixed enumeration of 4-space, and one has already constructed a collection L of $|k|$ -many (fewer than c) lines. If p is in the union of L there is nothing to do. Otherwise one need only find a unit tangent vector u at p so that the line through p in direction u is (a) disjoint from each line in L and (b) ultraskew to every p_i