Re: Identities in an algebra

Source: http://sci.tech−archive.net/Archive/sci.math.research/2005−02/0075.html

---

From: Bill Dubuque (*wgd_at_nestle.csail.mit.edu*)
Date: 02/13/05

`Date: 13 Feb 2005 11:02:29 −0500`

Kevin Buzzard <buzzard@imperREMial.aOVEc.uk> wrote:
>
> *A student of mine is finishing up writing his thesis; if he can*
> *prove that a certain diagram commutes then he will be done!*
>
> *We spent some time yesterday trying to work out what needs to be done.*
> *The question below isn't equivalent (the original question is about*
> *p−adic Siegel modular forms), but it suffices to finish the argument.*
> *The question below is basically equivalent to a messy combinatorial*
> *identity that nowadays could be checked by computer, but there is*
> *probably a neater way of finishing the job. Can anyone help?*
>
> *Let L be the 4−dimensional Lie algebra with basis D,E,F,G, and*
> *satisfying*
>
> *[D,E]=F, [D,F]=G, [D,G]=0,*
> *[E,F]=[E,G]=[F,G]=0*
>
> *(this is a Lie algebra; for example D could be d/dX*
> *and E could be X^2 d/dY acting on R[X,Y])*
>
> *Is it true that for all integers a,b>=0 the following identity*
> *is true in the universal enveloping algebra of L:*
>
> *a a+b a+2b b b a+2b a+b a*
> *D E D E = E D E D*

With D = d/dX it suffices to verify the following

 a 2a+2b a+2b 2b 2b a+2b 2a+2b a
D X D X = X D X D

This is easily verified by evaluating at X^n, i.e.

 (n+a+2b)..(n+2b+1) (n+2b)..(n−a+1) X^(n+2b)
= (n+a+2b) ... (n+1) n(n−1)..(n−a+1) X^(n+2b)

−−Bill Dubuque