Re: Identities in an algebra

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Bill Dubuque <wgd@nestle.csail.mit.edu> wrote:

> With D = d/dX it suffices to verify the following
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> a 2a+2b a+2b 2b 2b a+2b 2a+2b a
> D X D X = X D X D

Setting D=d/dX and E=X^2 gives a representation of the universal enveloping algebra, and you're checking that the identity I want is true in the image. But I don't think that the representation is injective; it sends F to 2X and G to 2, so it sends 2EG−F^2 to zero. On the other hand setting D=d/dX and E=X gives another representation of the algebra in which 2EG−F^2 is not sent to zero, so 2EG−F^2 is not zero in the algebra itself. Hence I don't see why it suffices to do this, unfortunately.

I can quite believe that an argument of this nature might work though. Proving that the image of the identity in a representation is zero seems to frequently amount to verifying combinatorial identities; the example above results in a rather simple identity but I am not so sure that it's strong enough to prove what I need. I know of more complicated representations which I can prove are injective; however the resulting combinatorial identities are also quite complicated (double sums of products of 6 binomial coefficients!). I am sure that one could prove such identities nowadays following Zeilberger et al, but I didn't really want to go down that path, I am still optimistic that there is a simpler solution [At the end of the day the real reason I don't want to push this approach through is that the identities above are in some sense a theorem (hopefully) about Sp_4, and I would one day like to prove analogous ones for an arbitrary reductive group, when brute force calculations would be doomed.]

Kevin