Lie groups that are not matrix groups

Source: http://sci.tech−archive.net/Archive/sci.math.research/2005−03/0149.html

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Date: 03/29/05

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Date: Tue, 29 Mar 2005 02:00:07 +0000 (UTC)
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So in the book Representation Theory by Fulton and Harris, they're classifying Lie algebras of dimension 3 in one of their beginning chapters. It made me want to ask some questions about Lie groups which are not matrix Lie groups, i.e. have no faithful finite dimensional representations.

Let me briefly describe the stuff in the text. They get to the real Lie algebra [X,Y]=[X,Z]=0, [Y,Z]=X, and goes on to exponentiate some representation of the algebra to get the group G of upper triangular 3x3 matrices with 1s along the diagonal. This group has center R, matrices with the only nonzero entry in the upper corner. To get other Lie groups with some algebra, they take the quotient of G/N with N any discrete normal subgroup, which must therefore be central. Z being a discrete normal subgroup, G/Z is the only other group with this algebra, presumably G/nZ will be isomorphic.

Then he proves that this group has no faithful irreps: if it did, the center, S^1, would be proportional to the identity in that irrep, and we would have [Y,Z]~1, but tr[Y,Z]=0.

My first question is about this proof. I can't understand why this proof doesn't apply to the covering group G (which clearly does have finite dimensional faithful reps). I think its center R should also be proportional to the identity in an irrep. The difference between the two cases is that S^1 is compact, while R is not. But so what? Did I miss the theorem in the book where he showed that a compact central subgroup must be proportional to the identity? Does not Schur's lemma apply to anything that commutes with the whole group?

He also talks about how the tower of covers of SL(2) are not matrix groups. And I seem to recall once hearing that the universal cover of GL(n) is not a matrix group. These Lie groups that are not matrix groups seem a bit mysterious. Is there a general way to find groups who are not matrix groups? Or turn one that is not into one that is? What are some others? I think I've heard that the universal cover of GL(n) is not. Is that right? What about Spin(n)? The Spin(n)s that I've met have been, but I've never seen a general case.

thanks
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