Re: Separable Linear Order

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- *From*: William Elliot <marsh@xxxxxxxxxxxxxxxxxx>
- *Date*: Mon, 1 Aug 2005 00:19:13 −0700

From: Fred Galvin <fred.galvin@xxxxxxxxx>
Newsgroups: sci.math.research
Subject: Re: Separable Linear Order

fred.galvin@xxxxxxxxx wrote:
> William Elliot wrote:
> > Given a linear order L with the usual interval topology,
> > how does one show when L is separable, L is Lindelof?
>
> Let C be any open cover of L; we may assume that the members of C are
> open intervals (a, b). Define an equivalence relation on L so that,
> when x < y, the points x and y are equivalent just in case the closed
> interval [x, y] is covered by countably many members of C. Each
> equivalence class is open, since it is the union of some subcollection
> of C. So the equivalence classes form a collection of disjoint nonempty
> open subsets of L. Since L is separable, it follows that there are only
> countably many equivalence classes. To finish the proof, we have to
> show that each equivalence class is covered by countably many members
> of C.
>
> >From the separability of L, it follows that L cannot have a subset of
> order type omega_1. Hence each equivalence class must have a countable
> cofinal subset, and (dually) a countable coinitial subset. Hence each
> equivalence class is the union of countably many closed intervals. By
> the definition of the equivalence relation, each closed interval
> (contained in an equivalence class) is covered by countably many
> members of C. That does it, I think.

> PS. Note that, instead of assuming that L is separable, you can get
> by with the weaker assumption that L satisfies the "countable chain
> condition", i.e., every collection of disjoint open sets is
> countable.

Yes
separable ==> ccc (countable chain condition)

and your assertion is correct because
linear order topologies are monotonically normal and
ccc, monotonically normal ==> (hereditarily) Lindelof

A space (S,tau) is monotonically normal (MN) when
for all x, open U nhood x, some open mu(x,U) nhood x
with for all x,y, open U,V,
x in U, y in V, nonnul mu(x,U) /\ mu(y,V) ==> x in V or y in U

The partial map mu:Sxtau -> tau is called a normality (for S).
Monotonically normal is hereditary and implies completely normal.
Metric spaces are MN.

I have tried to adapt your proof of
separable linear order topology ==> Lindelof
or
ccc, linear order topology ==> Lindelof
to
separable, monotonically normal ==> Lindelof
with an eye to
ccc, monotonically normal ==> Lindelof.

But I have netted a bankruptcy of ideas, unable to find any
remotely eligible equivalence relation for the first step.

Incidentally, a discrete subspace with cardinality kappa of a
monotonically normal space will yield a collection of pairwise disjoint
open sets with cardinality kappa. Thus for MN spaces, ccc and every
discrete subspace is countable, are equivalent.

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- ***References***:
  - ***Re: Separable Linear Order***
    - *From:* Fred Galvin

- Prev by Date: ***Re: Separable Linear Order***
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