Re: conformal map regular polygone into unit circle.

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- *From*: israel@xxxxxxxxxxx (Robert Israel)
- *Date*: 20 Sep 2005 14:38:33 −0400

In article <dgot3s$l7q$1@xxxxxxxxxxxxxxxxxxxxxxxxxx>, <vanesch@xxxxxx> wrote:

>I'm looking for the conformal mapping (using complex functions) that
>maps the unit circle (or the upper half plane) into a REGULAR polygon
>with n vertices. I know the Schwarz−Christoffel transformation for an
>ARBITRARY polygon, but that doesn't help me because the expression is
>way too complex to be integrated (I'm trying to find the mapping for a
>polygon with 120 vertices). I was hoping that the fact that the polygon
>is REGULAR would simplify the problem. I used the mapping on the unit
>circle in the S−C transform because out of the symmetry of the problem,
>that allowed me (I would guess) to fix the unknown images of the
>vertices: they should also be on a regular polygon. But nevertheless, I
>cannot solve the integral beyond n = 4.

The symmetry does help: using radial cuts, cut the circle into n equal
sectors and the n−gon into n equal triangles. If you can map a sector
to a triangle, with the radial sides going to the radial sides, then
the Schwarz reflection principle guarantees the rest falls into place.
Mapping the sector to the upper half−plane should be no problem.
You're left with mapping the upper half−plane to the triangle. The
Schwarz−Christoffel integral in this case is not elementary, but it
can be done, according to Maple, with a hypergeometric function:

int 1/(1−w^2)^(1−2/n) dw = w hypergeom([1/2, 1−2/n],[3/2],w^2)

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- ***References***:
  - ♦ ***conformal map regular polygone into unit circle.***
    - ◊ *From:* vanesch

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