Re: conformal map regular polygone into unit circle.

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- *Date*: 20 Oct 2005 10:15:00 −0400

On 20 Sep 2005 vanesch@xxxxxx wrote:

>
> Hi All,
>
> I'm looking for the conformal mapping (using complex functions) that
> maps the unit circle (or the upper half plane) into a REGULAR polygon
> with n vertices. I know the Schwarz−Christoffel transformation for an
> ARBITRARY polygon, but that doesn't help me because the expression is
> way too complex to be integrated (I'm trying to find the mapping for a
> polygon with 120 vertices). I was hoping that the fact that the polygon
> is REGULAR would simplify the problem. I used the mapping on the unit
> circle in the S−C transform because out of the symmetry of the problem,
> that allowed me (I would guess) to fix the unknown images of the
> vertices: they should also be on a regular polygon. But nevertheless, I
> cannot solve the integral beyond n = 4.
>
> Any hints, papers, books, ... welcome.
>
> thanks,
> Patrick.
>
>
Hi Patrick,

I think your idea with symmetry is excellent. It seem to me, that the
integral is generally a hypergeometric function.

The SC−transformation
(see e.g. Frank, v. Mises Differential−und Integralgleichungen der Physik)

z= G(x) =
int_0^x (1−e^(pi i a) u)^a (1−e^(2 pi i a) u)^a (1−e^(3 pi i a) u)^a
....(1−e^(n pi i a) u) du =
int_0^x (1−u^n)^a du

with a=−2/n

transforms a circle in the x−plane into a regular n−gon in the z−plane. One has obviously G(e^(pi i a) x) = e^(pi i a) G(x). This means that the image is unchanged if it is rotated by an angle 2 pi/n.

Substituting v=u^n shows that the function G(x) is an incomplete Beta−function which can also be expressed in terms of a hypergeometric function (Abramowitz, Stegun Handbook of Mathematical Functions)

G(x)=1/n B_{x^n}(1/n, 1−2/n)
=x F(1/n,2/n;1+1/n;x^n)

Willi

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