This Week's Finds in Mathematical Physics (Week 260)

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This Week's Finds in Mathematical Physics (Week 260)
John Baez

Since it's Christmas Eve, I thought I'd list some free books you can download. I'm a big fan of giving the world presents... and I'm not the only one.

But first, this week's nebulae! Here's one called the Retina:

1) Retina Nebula, Hubble Heritage Project,
http://heritage.stsci.edu/2002/14/

This is actually a tube of ionized gas about a quarter of a light−year across and one light−year long. It's a planetary nebula produced by a dying star. If you zoom in and look closely, you can see this star lurking in the middle, now a mere white dwarf.

The blue light is the most energetic, so it's really hot where you see blue. This blue light comes from singly ionized helium − helium where one electron has been knocked off. The green light is a bit less energetic: that's from doubly ionized oxygen. The red light comes from even cooler regions: that's from singly ionized nitrogen.

You can also see a lot of "dust lanes" in this photo. They're beautiful. But what creates them?

Apparently, when the fast−moving glowing hot gas from the star crashes into the invisible gas in the surrounding interstellar space, the boundary gets sort of crumpled, and these dust lanes form. It's vaguely similar to the puffy surface of a cumulus cloud. But here the mechanism is different, because it involves a "shock wave": the hot gas is moving faster than the speed of sound as it hits the cold gas!

This effect is called a "Vishniac instability", since in 1983, the astrophysicist Ethan Vishniac showed that a shock wave moving in a sufficiently compressible medium would be subject to an instability of this sort, growing as the square root of time. I've never seen how Vishniac's calculations work, so the mathematics underlying this beautiful phenomenon will have to wait for another day.

Note that this planetary nebula, like the others I've shown you, is far from spherically symmetric. Astrophysicists used to pretend stars were spherically symmetric. But, that's a bad approximation whenever anything really exciting happens... just like in the old joke where the punchline is "consider a spherical cow".

As I said, the Retina Nebula is actually shaped like a tube. Viewed from either end, this tube would look very different − probably like the Ring Nebula:

2) Ring Nebula, Hubble Heritage Project,
http://heritage.stsci.edu/1999/01/

This is one light−year across. Again we see He II blue light with a wavelength of 4686 angstroms, then O III green light at 5007 angstroms, then N II red light at 6584 angstroms. You can also see the white dwarf as a tiny dot in the center; it's about 100,000 kelvin in temperature.

(In case you're wondering, an "angstrom" is an obsolete but popular unit of distance, equal to 10^{−10} meters. Just like the "parsec", it's a sign that astronomy is an old science. Anders Jonas Angstrom was one of the founders of spectroscopy, back around 1860. Archaic conventions may also explain why single ionized helium is called "He II", and so on. Maybe the number zero hadn't fully caught on.)

Next: free books!

At least around here, Christmas seems to be all about buying stuff and giving it away. Giving is good. But I think gifts have more soul if you make them yourself. This is one of the great things about the internet: it lets us create things and give them to *everyone in the world* − or more precisely: everybody who wants them, and nobody who doesn't.

In this spirit, here's a roundup of free books on math and physics: gifts from their authors to you. There are lots out there. I'll only list a few. For more, try these sites:

3) George Cain, Online Mathematics Textbooks,
http://www.math.gatech.edu/~cain/textbooks/onlinebooks.html

4) Free Online Mathematics Books,
http://www.pspxworld.com/book/mathematics/

5) Alex Stefanov, Textbooks in Mathematics,
http://users.ictp.it/~stefanov/mylist.html or (with annoying ads,
but more permanent) http://us.geocities.com/alex_stef/mylist.html

Despite its title, Stefanov's excellent site includes a lot of books on physics. I can't find lists *specifically* devoted to free physics books, but there are a lot out there − including a lot on the arXiv.

Anyway, let's dive in!

What if you're dying to learn physics, but don't know where to start? Start here:

6) Christoph Schiller, Motion Mountain: The Adventure of Physics,
available free online at http://www.motionmountain.net/

It's an enormous feast of ideas − romantic, wildly ambitious, and still not finished at 1459 pages. Using a bare minimum of math, it conveys an enormous amount of physics, all focused on the question "what is motion?" This question is very deep. We have made tremendous progress towards answering, but are nowhere near done.

The curious title is explained near the beginning:

The quest to understand motion in all its details and limitations can be pursued behind a desk, with a book, some paper and a pen. But to make the adventure more vivid, this text uses the metaphor of a mountain ascent. Every step towards the top corresponds to a step towards higher precision in the description of motion. In addition, with each step the scenery will become more delightful. At the top of the mountain we shall arrive in a domain where 'space' and 'time' are words that have lost all meaning and where the sight of the world's beauty is overwhelming and unforgettable.

Inspiring words. But to dig deeper into such mysteries, you'll eventually need to learn a bunch of math. Do you remember what Victor Weisskopf said when a student asked how much math a physicist needs to know? "More." This can be scary when you're just getting started. What if you don't know calculus, for example?

Simple: learn calculus! This book is a classic − and it's free:

7) Gilbert Strang, Calculus, Wellesley−Cambridge Press, Cambridge,
1991. Also available at
http://ocw.mit.edu/ans7870/resources/Strang/strangtext.htm<

It really explains things clearly. I may use it the next time I teach calculus. We professors need to quit making our students buy expensive textbooks, and switch to free online books! We could join forces and make wiki textbooks that are a lot better and more flexible than the budget−busting, back−breaking mammoths we currently inflict on our kids. But there are already a lot of good texts available free online.

Or: what if you know calculus, but you're still swimming through the undergraduate sea of differential equations, Fourier transforms, matrices, vectors and tensors? Then this should be really helpful:

8) James Nearing, Mathematical Tools for Physics, available at
http://www.physics.miami.edu/~nearing/mathmethods/

Unlike the usual dry and formal textbook, it reads like a friendly uncle explaining things in plain English, trying to cut through the red tape and tell you how to actually think about this stuff.

For example, on page 3 he introduces the hyperbolic trig functions:

Where do hyperbolic functions come from? If you have a mass in equilibrium, the total force on it is zero. If it's in *stable* equilibrium then if you push it a little to one side and release it, the force will push it back to the center. If it is *unstable* then when it's a bit to one side it will be pushed farther away from the equilibrium point. In the first case, it will oscillate about the equilibrium position and the function of time will be a circular trigonometric function − the common sines or cosines of time, A cos(wt). If the point is unstable, the motion will be described by hyperbolic functions of time, sinh(wt) instead of sin(wt). An ordinary ruler held at one end will swing back and forth, but if you try to balance it at the other end it will fall over. That's the difference between cos and cosh.

He goes into more detail later, after introducing the complex numbers. This book also features some great animations of Taylor series and Fourier series.

There are free online books at all levels... so let's soar a bit higher. How about if you're a more advanced student trying to learn general relativity? Here you go:

9) Sean M. Carroll, Lecture Notes on General Relativity, available as arXiv:gr−qc/9712019

How about quantum field theory? Then you're in luck − there are *two* detailed books available online:

10) Warren Siegel, Fields, available as arXiv:hep−th/9912205

10) Mark Srednicki, Quantum Field Theory, Cambridge U. Press, Cambridge, 2007. Also available at
http://www.physics.ucsb.edu/~mark/qft.html

Or what about algebraic topology? Again you're in luck, since you can read both Allen Hatcher's gentle introduction and Peter May's high−powered "concise course":

11) Allen Hatcher, Algebraic Topology, Cambridge U. Press, Cambridge, 2002. Also available at http://www.math.cornell.edu/~hatcher/AT/ATpage.html

12) Peter May, A Concise Course in Algebraic Topology, U. of Chicago Press, Chicago, 1999. Also available at http://www.math.uchicago.edu/~may/CONCISE/ConciseRevised.pdf

May has a lot of more advanced topology books available at his website, too − like this classic, where he used operads to solve important problems involving loop spaces:

13) Peter May, The Geometry of Iterated Loop Spaces, Lecture Notes in Mathematics 271, Springer, Berlin, 1972. Also available at http://www.math.uchicago.edu/~may/BOOKS/gils.pdf

Or say you want to learn about vector bundles and how they show up in physics, from the basics all the way to fancy stuff like D−branes and K−theory? Try this − it's a great sequel to Husemoller's classic intro to fiber bundles:

14) Dale Husemoller, Michael Joachim, Branislav Jurco and Martin Schottenloher, Basic Bundle Theory and K−Cohomology Invariants, Lecture Notes in Physics 726, Springer, Berlin, 2008. Also available at http://www.mathematik.uni−muenchen.de/~schotten/Texte/978−3−540−74955−4_Book_LNP726.pdf

The list goes on and on! The American Mathematical Society will give you books for free if you prove that you're not a robot by solving a little puzzle:

15) American Mathematical Society, Books Online By Subject, http://www.ams.org/online_bks/online_subject.html

Apparently they don't want robots learning advanced math and putting us professors out of business by teaching with more charisma and flair. (By the way: make sure to let them put cookies on your webbrowswer, or they'll send you an endless succession of these puzzles, without explaining why!

Since James Dolan and I plan to explain symmetric groups and their Hecke algebras in our online seminar, this particular book from the AMS caught my eye:

16) David M. Goldschmidt, Group Characters, Symmetric Functions, and the Hecke Algebra, AMS, Providence, Rhode Island, 1993. Also available as http://www.ams.org/online_bks/ulect4/

Since we're also struggling to understand the Langlands program,
this looks good too:

17) Armand Borel, Automorphic Forms, Representations, and L−functions, AMS, 2 volumes, Providence, Rhode Island, 1979. Also available at http://www.ams.org/online_bks/pspum331/ and http://www.ams.org/online_bks/pspum332/

It's a serious collection of expository papers by bigshots like Borel, Cartier, Deligne, Jacquet, Knapp, Langlands, Lusztig, Tate, Tits, Zuckerman, and many more.

"Motives" are the mysterious virtual building blocks that algebraic varieties are built from. If you're ready to learn about motives − I'm not sure I am − try this:

18) Marc Levine, Mixed Motives, AMS, Providence, Rhode Island, 1998. Also available at http://www.ams.org/online_bks/surv57/

Or, if you're interested in using category theory to make analysis clearer and more beautiful, try this:

19) Andreas Kriegl and Peter W. Michor, The Convenient Setting of Global Analysis, AMS, Providence, Rhode Island, 1997. Also available at http://www.ams.org/online_bks/surv53/

The focus is on getting and working with a "convenient category" of infinite−dimensional manifolds. The idea of a "convenient category" goes back to topology: at some point, people realized they wanted this property to hold:

C(X x Y, Z) = C(X, C(Y,Z))

Here C(X,Y) is the space of maps from X to Y. So, the equation above − really an isomorphism − says that a map from X x Y to Z should correspond to a map from X to C(Y,Z). A category with this property is called "cartesian closed". While it's probably not obvious at first, this property is so wonderful that people threw out the category of topological spaces and continuous maps and replaced it with a slightly different one, to get this to hold.

Another sort of "convenient category" for differential geometry uses infinitesimals. Again, you can learn about this in a free book:

20) Anders Kock, Synthetic Differential Geometry, Cambridge U. Press, Cambridge, 2006. Also available at http://home.imf.au.dk/kock/

This category is not just cartesian closed − it's a topos!

If you don't know what a topos is, never fear − more free books are coming to your rescue:

21) Robert Goldblatt, Topoi, the Categorial Analysis of Logic,
Dover, 1983. Also available at
http://historical.library.cornell.edu/cgi−bin/cul.math/docviewer?did=Gold010

22) Michael Barr and Charles Wells, Toposes, Triples and Theories,
Springer, Berlin, 1983. Also available at
http://www.case.edu/artsci/math/wells/pub/ttt.html

The first one is so gentle it makes a good introduction to category theory as a whole. The second scared the bejeezus out of me for a decade, but now I like it.

I like Jordan algebras, so I was also pleased to see this classic offered for free at the AMS website:

23) Nathan Jacobson, Structure and Representations of Jordan Algebras,
AMS, Providence, Rhode Island, 1968. Also available at
http://www.ams.org/online_bks/coll39/

Fans of exceptional Lie algebras will like the last two chapters, on "connections with Lie algebras" and "exceptional Jordan algebras".

Speaking of Lie algebras, I'd never seen this textbook before:

24) Shlomo Sternberg, Lie Algebras,
http://www.math.harvard.edu/~shlomo/docs/lie_algebras.pdf

It's a somewhat quirky introduction, not for beginners I think, but it features some nice special topics: character formulas, the Kostant Dirac operator, and a detailed study of the center of the universal enveloping algebra.

This intro to Lie groups is also a bit quirky, but if you like Feynman diagrams or spin networks, it's irreplaceable:

25) Predrag Cvitanovic, Birdtracks, Lie's, and Exceptional Groups,
available at http://www.nbi.dk/GroupTheory/

One of the great things about this book is that it classifies simple Lie groups according to their "skein relations" − properties of their representations, written out diagrammatically. In so doing, Cvitanovic realized that there's a "magic triangle" containing all the exceptional Lie groups. This subsumes the "magic square" of Freudenthal and Tits, which I discussed in "week145" and my octonion webpages.

This idea of Cvitanovic is closely related to the "exceptional series" of Lie groups − a pattern whose existence was conjectured by Deligne.

Originally this was going to be the topic of this Week's Finds, but

I'm a bit too tired to explain it now. Still, I love the idea. It's an oxymoron, since the exceptional groups were defined as those that don't fit into any series. But, it makes sense!

To see the exceptional series, it helps to do a mental backflip called "Tannaka−Krein duality", where you focus on the category of representations of the Lie group, instead of the group itself. Then, draw the morphisms in that category as diagrams, like Feynman diagrams! Then see what identities these diagrams satisfy. New patterns leap out: new series unify what had been "exceptions".

So, I urge all fans of exceptional mathematics, diagrams, and categories to look at these:

26) Pierre Deligne, La serie exceptionnelle des groupes de Lie, C. R. Acad. Sci. Paris Ser. I Math 322 (1996), 321−326.

Pierre Deligne and R. de Man, The exceptional series of Lie groups II, C. R. Acad. Sci. Paris Ser. I Math 323 (1996), 577−582.

Pierre Deligne and Benedict Gross, On the exceptional series, and its descendants, C. R. Acad. Sci. Paris Ser. I Math 335 (2002), 877−881. Also available as http://www.math.ias.edu/~phares/deligne/ExcepSeries.ps

27) Pierre Vogel, Algebraic structures on modules of diagrams, 1995. Available at http://www.institut.math.jussieu.fr/~vogel/ or http://citeseer.ist.psu.edu/469395.html

The universal Lie algebra, 1999. Available at http://www.institut.math.jussieu.fr/~vogel/

Vassiliev theory and the universal Lie algebra, 2000. Available at http://www.institut.math.jussieu.fr/~vogel/

For a good overview of what Landsberg and Manivel have done in this field, try:

28) J. M. Landsberg and L. Manivel, Representation theory and projective geometry, 2002. Available at arXiv:math/0203260.

For more details, try these:

29) J. M. Landsberg and L. Manivel, The projective geometry of Freudenthal's magic square, J. Algebra 239 (2001), 477−512. Also available as arXiv:math/9908039.

J. M. Landsberg and L. Manivel, Triality, exceptional Lie algebras and Deligne dimension formulas, Adv. Math. 171 (2002), 59−85. Also available as arXiv:math/0107032.

J. M. Landsberg and L. Manivel, Series of Lie groups, available as arXiv:math/0203241.

For even more, try this:

30) Bruce Westbury, References on series of Lie groups,
http://www.mpim−bonn.mpg.de/digitalAssets/2763_references.pdf

This stuff has been on my mind recently, since I've been working on exceptional groups and grand unified theories with my student John Huerta. Also, my friend Tevian Dray has a student who just finished a thesis on a related topic:

31) Aaron Wangberg, The structure of E6, available as arXiv:0711.3447.

In a nutshell: E6 is secretly SL(3,O). Octonions rock!

Happy holidays. Keep learning cool stuff.

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Quote of the Week:

If nature has made any one thing less susceptible than all others of exclusive property, it is the action of the thinking power called an idea, which an individual may exclusively possess as long as he keeps it to himself; but the moment it is divulged, it forces itself into the possession of every one, and the receiver cannot dispossess himself of it. Its peculiar character, too, is that no one possesses the less, because every other possesses the whole of it.

Thomas Jefferson

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Previous issues of "This Week's Finds" and other expository articles on mathematics and physics, as well as some of my research papers, can be obtained at

http://math.ucr.edu/home/baez/

For a table of contents of all the issues of This Week's Finds, try

http://math.ucr.edu/home/baez/twfcontents.html

A simple jumping−off point to the old issues is available at

http://math.ucr.edu/home/baez/twfshort.html

If you just want the latest issue, go to

http://math.ucr.edu/home/baez/this.week.html

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