Re: −− A sequence or real numbers

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On Feb 13, 5:30 pm, c...@xxxxxxxxxxxxxxx wrote:

> Does anyone know, or at least have a clue on the following question ?
>
> Let (b_n)_n be the sequence defined by :
>
> b_1=1/4 , b_2=1/16 and
>
> b_{n+1}=b_1 b_n+b_2 b_{n−1}+ ... +b_k b_{n−k+1}+ ... +b_1 b_n
>
> Ex: b_3=b_1 b_2+b_2 b_1=1/32
> b_4=b_1 b_3+b_2 b_2+b_3 b_1=5/256 etc.
>
> Finally, let
>
> X_n=b_n . n^{3/2}
>
> A computer evaluation hints to the fact that (X_n) is decreasing to a
> limit approx equal to 1.141...
>
> Proof (?)
>
> Note: the initial values (i.e. b_1,b_2) seems to be essential. Any
> change, and (X_n)_n appears
> to behave chaotically. [again, computer says so]
>
> Any information is warmly welcome.
> Thanks
> cosv

Define a (formal) series putting

f = sum_{n >= 1} b_n t^n

One can easily see this actually has a positive convergence
radius. Your recurrence relation means that

f^2 = f − 1/4

Solving this tells us that

$f = (1 - \sqrt{1 - t}) / 2$

Therefore you can compute the b_n exactly by
figuring out the derivatives of f:

$b_n = f^{(n)}(0) / n!$

The derivatives at zero have a simple form,
which will let you find what you want.

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