

# Dual norms

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(In the following, we may work on  $\mathbb{R}^n$ , though the same questions arise in other spaces.)

Consider two norms  $\|\cdot\|_a$  and  $\|\cdot\|_b$ . Then

$$\phi: v \rightarrow (|v|_a^p + |v|_b^p)^{1/p}$$

is itself a norm. This is a special case of a much more general statement – a norm of norms is a norm.

Question: what is the dual norm of  $\phi$ ?

For example, what is the dual norm of

$$v \rightarrow (c_1 |v|_1^2 + c_2 |v|_2^2)^{1/2},$$

where  $c_1, c_2$  are positive numbers? (I do not know the answer even for  $c_1=1, c_2=2$ .)

Harald

PS. The answer is not  $((1/c_1) |v|_{\infty}^2 + (1/c_2) |v|_2^2)^{1/2}$ , as one might think at first.

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