Things get nicer at some huge number?

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Mathematicians have long noticed that in many fields, theorems have exceptions for small integers. Some theorems for compact differentiable manifolds can be proved for n bigger than 4, but things go haywire for 1, 2, 3, 4, especially 4. The finite simple groups have all been classified as being in one of several infinite families, with a finite list of exceptions, the largest being of order less than 10^54. (Well, that is small relative to most numbers!) The largest exceptional Lie group is a manifold of dimension 248. The prime counting function finally decides to be bigger than the logarithmic integral somewhere around 10^316. The smallest Perrin pseudoprime that is not a prime is 271,441.

Perhaps math gets better behaved for very large integers. This suggests a conjecture:

THE BIG NUMBER CONJECTURE CONJECTURE (sic)
If P(n) is a mathematical statement with one free variable n that ranges over the positive integers, then there is a number B_P depending only on the form of P with the property that, in order to prove that P(n) is true for all positive integers, it is sufficient to prove P(n) for all positive integers less than B_P.

Remarks:

a) This is precisely a conjecture that a meaningful conjecture exists.
b) The BNC is not a proper conjecture until I define "mathematical statement" precisely. Anyway, it may be true for some forms of statements and not others.
c) B_P has to depend on P because you could replace P(n) by P(f(n)) where f(n) is some slow growing function, such as the greatest integer in log log n.
d) But the dependence of B_P on P must be on the FORM of P in some sense (number of quantifiers or some such thing). Otherwise the conjecture is trivially true.

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