Re: p−Lower central series, relevance?

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- *From*: Derek Holt <mareg@xxxxxxxxxxxxx>
- *Date*: Mon, 24 Nov 2008 14:00:02 +0000 (GMT)

On 23 Nov, 09:00, combar...@xxxxxxxxx wrote:

> Given a group G, the p−lower central series is defined by:
>
> $P_0(G)=G$;
> $P_{i+1}(G)=[G,P_i(G)]P_i(G)$.
>
> Could anyone please tell me the relevance of this subgroups, or a good reference. I found for example that the automorphism of a finite p−group can be computed inductively using this groups.
>
> Thanks.

You have a typo in your definition. It should be:

$P_{i+1}(G)=[G,P_i(G)]P_i(G)^p$.

So P_{i+1} is the smallest normal subgroup of G such that $P_i(G)/P_{i+1}(G)$ is elementary abelian and central in G/P_{i+1}.

I would guess that it arises most frequently in computational group theory, particularly when computing with finite p−groups, where it is often the most convenient central series for computational purposes.

There is a very efficient algorithm known as the nilpotent quotient, or p−quotient algorithm, which takes a group G defined by a finite presentation as input, and successively computes the terms of its lower p−central series. This has been used, for example, to compute the restricted Burnside groups − the most recent being R(2,7), which has order 7^{20416}, and took about 1 CPU−year to construct.

As a general reference and pointer to other references, I would suggest the book "Handbook of Computational group Theory" by D F Holt, B Eick, and E.A. O'Brien, p.355 ff.

Derek Holt.

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