

Fibonacci[1,000,000,000] contains 208,987,640 decimal digits

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"Ed Pegg Jr" <edpegg@gmail.com> wrote in comp.soft-sys.math.mathematica:

> For finding the number of digits in any sufficiently large Fibonacci
> number in base 10, let $k = (\text{ArcCsch}[2])/\text{Log}[10]$. The number of
> digits in $\text{Fibonacci}[n]$ is $\text{Round}[n k]$.
>
> $k \sim 0.20898764024997873376927208923755541682245923991821$
>
> $\text{Fibonacci}[10^9]$ has $10^9 k \sim 208987640$ digits.

Thank you for confirming this result. It means, that the number mentioned in Sloane's sequence A068070 (number of digits of the 10^n -th Fibonacci number), like some other numbers in this sequence, are wrong.

N. Sloane get now a comment to A068070 with the corrected sequence {0,2,21,209,2090,20899,208988,2089877,20898764,208987640,2089876402,20898764025,208987640250,2089876402500,20898764024998,208987640249979,2089876402499787,20898764024997873,208987640249978734,...}.

In addition, here is a fast way to find the n -th Tribonacci number:

```
rho=1/3*((19+3*Sqrt[33])^(1/3)+(19-3*Sqrt[33])^(1/3)+1);  
triboappr[n_]:=N[(rho-1)/(4rho-6)*rho^(n-3),3000];
```

For getting the digits of the 10^n -th Tribonacci number, we could try a constant $k_2 = 0.264649...$ (similar to the k above), or quick and dirty

```
Table[MantissaExponent[triboappr[10^i]][[2]],{i,1,7}]
```

and get

```
{2, 26, 264, 2646, 26464, 264649, 2646494,...}
```

sci.math.symbolic: Fibonacci[1,000,000,000] contains 208,987,640 decimal digits

Until now, this sequence was not in Sloane's database and is sent there.

Greetings from Hamburg,

Michael Taktikos

P.S. Because the last mail I sent to comp.soft-sys.mathematica somehow got lost, this mail will appear in sci.math.symbolic, too.